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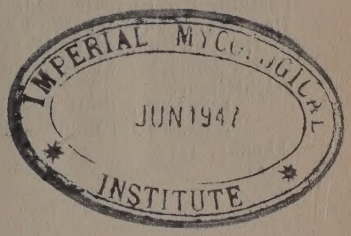

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MATHEMATICS

THE RATIONAL ANAMORPHOSIS OF EQUATIONS

By P. V. NIKOLAEV

(Communicated by A. N. Kolmogoroff, Member of the Academy, 27. V. 1944)

In the present paper we consider the question on the anamorphosibility of irreducible algebraic equations

$$F(t_1,\ t_2,\ t_3)=0 \tag{1}$$

i. e. the question on the existence of such a polynomial

$$\Phi(t_1,\ t_2,\ t_3)\equiv\psi_1(t_2,\ t_3)\,\psi_2(t_3,\ t_1)\,\psi_3(t_1,\ t_2)\,F(t_1,\ t_2,\ t_3) \tag{1'}$$

which admits of a rational anamorphosis and, consequently, may be identically represented in the form of a certain Massau determinant

$$\Phi(t_1,\ t_2,\ t_3)\equiv\left|f_{i1}(t_i)\ f_{i2}(t_i)\ f_{i3}(t_i)\right| \tag{1''}$$

where $f_{ik}(t_i)$ $(i,\ k=1,\ 2,\ 3)$ are polynomials in t_i.

The conditions, under which a given polynomial $\Phi(t_1,\ t_2,\ t_3)$ admits of an anamorphosis (*i. e.* is a Massau polynomial) were already considered by the author; the question on the uniqueness of the anamorphoses was also considered (1), and an algorithm of anamorphosibility was deduced (2). For the equation (1) the question on the uniqueness of admittable anamorphoses was also investigated (3).

Moreover, the author has found the conditions for the case where this equation is normal and at least binarily symmetrical.

Let (1) be the Massau equation (3) with the canonical (2) in t_1 decomposition

$$F(t_1,\ t_2,\ t_3)\equiv\sum_{i=1}^{r_1}\varphi_{1i}F_{1i}^{(2)} \tag{2}$$

and $t_1=t_{11}$ the general value of the variable t_1.

If $\varphi_1(t_1,\ t_{11})$ is the greatest common divisor of the polynomials

$$\varphi_{i1}(t_1)\ \varphi_{1l}(t_{11})-\varphi_{1l}(t_1)\ \varphi_{1i}(t_{11})\quad(i,\ l=1,\ldots,\ r_1;\ i\neq l). \tag{3}$$

then the equation

$$\varphi_1(t_1,\ t_{11})=0 \tag{4}$$

defines the involution group of the variable t_1 determined by its value $t_1=t_{11}$.

The theorem stated enables us to find the involution equation of the variable t_1 and, consequently, to norm the equation (1). Namely, if the polynomial

$$\varphi_1(t_1, t_{11}) = A_0(t_1)\, t_{11}^{\mu_1} + A_1(t_1)\, t_{11}^{\mu_1 - 1} + \ldots + A_{\mu_1}(t_1) \tag{5}$$

is the greatest common divisor of the polynomials (3), then for involution equation of the variable t_1 may be taken any of the equations

$$\tau_1 = \frac{A_1(t_1)}{A_{\mu_1}(t_1)} \quad (i = 1, \ldots, \mu_1 - 1) \tag{6}$$

if only the ratio $A_i(t_1)/A_{\mu_1}(t_1)$ is not a constant.

If now (1) is a certain irreducible algebraic equation (possibly not a Massau equation), then the equation (4) will be called the norming equation by t_1 for the equation (1), and the equation (1) itself will be called normed by t_1 in the case where the norming equation by t_1 is linear.

The normed (by all variables) equation obtained by eliminating the variables t_1, t_2, t_3 from the given equation (1) and the involution equations of the variables t_1, t_2, t_3 will be called the normal part of the equation (1).

A normed equation admitting of rational anamorphoses is evidently a normal Massau equation. Thus, the question is reduced to the anamorphosis of a normed equation.

The anamorphosis of equations without using the A-factor is completely investigated (²). But in the case where a normal equation admits of an anamorphosis with a common basis for the variables t_2 and t_3, it is always possible to make the equation symmetric, *i. e.* it is always possible to choose the constants a, b, c, d in such a way that the substitution

$$\tau_2 = -\frac{c\tau_2 + d}{a\tau_2 + b}. \tag{7}$$

reduces the normal Massau equation (1) of degree m_i with respect to t_i to a new Massau equation

$$F\left(t_1; -\frac{c\tau_2 + d}{a\tau_2 + b};\ t_3\right)(a\tau_2 + b)^{m_2} = F_1(t_1, t_2, t_3) = 0 \tag{8}$$

symmetric with respect to τ_2, τ_3. Thus, the question is reduced to the anamorphosis of symmetric (and normed) equations.

Further, it may be shown that a normal, symmetric with respect to t_2 and t_3 equation of non-zero genus with respect to t_1 admitting an A-factor possesses anamorphoses with identical scales for the variables t_2 and t_3. If, however, this equation is of zero genus with respect to t_1, and is not an equation of the third nomographical order, then the factor, of which it admits, will be of the form

$$\psi_1(t_2, t_3) = t_2 - t_3 \tag{9}$$

or of the form

$$\psi_1(t_2, t_3) = at_2t_3 + b(t_2 + t_3) + d \tag{10}$$

where a, b, d are certain constants.

Herewith it turns out that it is always possible by means of the substitution (7) to reduce any symmetric, normal equation with an A-factor of the form (10) to a symmetric equation with an A-factor of the form (9).

These facts enable us to indicate the following way of investigation of the rational anamorphosibility of the equation (1).

1. Find canonic decomposition of the polynomial

$$F(t_1, t_2, t_3) \equiv \sum_{i=1}^{r_1} \sum_{j=1}^{r_2} \sum_{k=1}^{r_3} a_{kji}\varphi_{1i}\varphi_{2j}\varphi_{3k} \tag{11}$$

If all the numbers r_1, r_2, r_3 are different and at least one of them exceeds three, then the equation (1) is not anamorphosible.

2. Find for every of the variables t_i the involution equations according to the formulae of the form (6), and norm the equation (1), reducing in this way the problem to a normed equation; the numbers r_1, r_2, r_3 are invariant with respect to the process of norming.

3. If neither of the numbers r_i exceeds three, then it is possible that the polynomial $F(t_1, t_2, t_3)$ is a Massau polynomial.

Leaving aside the elementary case of the equation of the third nomographical order, we may always assume that the canonic A-matrix $T^{(k)}$ possesses three columns.

Suppose for definiteness that the matrix $\overline{T}^{(1)}$ is taken. For $r_2=2$ the matrix $\overline{T}^{(1)}$ has the form:

$$\begin{array}{c|ccc} \overline{T}^{(1)} & f_{31} & f_{32} & f_{33} \\ \hline f_{21} & f_{11} & f_{12} & f_{13} \\ f_{22} & f_{14} & f_{15} & f_{16} \end{array} \tag{12}$$

and for $r_2=3$ the form

$$\begin{array}{c|ccc} \overline{T}^{(1)} & f_{31} & f_{32} & f_{33} \\ \hline f_{21} & f_{12} & f_{11} & f_{13} \\ f_{22} & f_{14} & f_{15} & f_{16} \\ f_{23} & f_{17} & f_{18} & f_{19} \end{array} \tag{13}$$

Herein $f_{2i} \equiv f_{2i}(t_2)$ and $f_{3i} \equiv f_{3i}(t_3)$, $(i=1, 2, 3)$, are the generatrices of the canonic, respectively by t_2, t_3, decompositions of the polynomial $F(t_1, t_2, t_3)$.

For the Massau polynomial $F(t_1, t_2, t_3)$ and only for it there exists a non-singular matrix

$$C=\|C_{ij}\| \tag{14}$$

such that the product of the matrix $\overline{T}^{(1)}$ by C yields the Massau matrix $M^{(1)}$

$$\overline{T}^{(1)}C=M^{(1)} \tag{15}$$

i. e. for $r_2=2$ a matrix of the form

$$M^{(1)}=\left\| \begin{array}{ccc} -\overline{f_{13}} & 0 & \overline{f_{11}} \\ \overline{f_{12}} & -\overline{f_{11}} & 0 \end{array} \right\| \tag{16}$$

and for $r_2=3$ a matrix of the form

$$M^{(1)}=\left\| \begin{array}{ccc} 0 & \overline{f_{13}} & -\overline{f_{12}} \\ -\overline{f_{13}} & 0 & \overline{f_{11}} \\ \overline{f_{12}} & -\overline{f_{11}} & 0 \end{array} \right\| \tag{17}$$

In the case of existence of the indicated matrix C, the anamorphosis of the polynomial $F(t_1, t_2, t_3)$ will be

$$F(t_1, t_2, t_3) \equiv \left| \begin{array}{ccc} \overline{f_{11}} & \overline{f_{12}} & \overline{f_{13}} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{array} \right| \tag{18}$$

where

$$f'_{3i} = \frac{1}{|C|} \sum_{j=1}^{3} C_{ji} f_{3j} \quad (i=1, 2, 3) \tag{19}$$

4. If the polynomial $F(t_1, t_2, t_3)$ of the normed equation (1) is not a Massau polynomial, then it is possible that the equation (1) admits of an A-factor. In this case it is possible to make the equation symmetric with

respect to at least a pair of the variables by means of substitutions of the form

$$t_i = -\frac{c\tau_i + d}{a\tau_i + b} \tag{20}$$

5. Suppose that the normed equation (1) is symmetric with respect only to the pair of variables t_2 and t_3. If now $r_1 > 3$, then this equation is not anamorphosible.

Let $r_1 = 3$. Then the equation

$$F(t_1, t_2, t_3) \equiv \sum_{k=0}^{m} t_3^k F_k^{(12)} = 0 \tag{2'}$$

will admit of an anamorphosis if and only if all the terms of the sequence

$$-F_{k-1}^{(12)} + t_2 F_k^{(12)} \quad (k = 0, \ldots, m+1) \tag{21}$$

are a linear combination of some three of them(4).

In this case the polynomial

$$\Phi(t_1, t_2, t_3) = (t_2 - t_3) F(t_1, t_2, t_3) \tag{22}$$

will be a Massau polynomial.

6. Suppose that the normed equation (1), symmetric with respect to t_2, t_3 and of zero genus with respect to t_1 does not admit of a direct anamorphosis, nor of an A-factor of the form (11).

Then it is anamorphosible if and only if there are such constants a, b, d that the substitution

$$t_2 = -\frac{b\tau_2 + d}{a\tau_2 + b} \tag{23}$$

reduces it to a new equation admitting of an A-factor

$$\psi_1(t_2, t_3) = \tau_2 - t_3. \tag{24}$$

7. Assume, finally, that the normed equation (2') is symmetric with respect to all three variables. If it is not an equation of zero genus, then it is anamorphosible in that and only in that case where in the sequence

$$-F_{j-1}^{(12)} + (t_1 + t_2) F_j^{(12)} - t_1 t_2 F_{j+1}^{(12)} \quad (j = 0, \ldots, m+1) \tag{25}$$

there are three terms such that every term of the sequence (25) is a linear combination of these three terms.

In this case the polynomial

$$\Phi(t_1, t_2, t_3) = (t_1 - t_2)(t_2 - t_3)(t_3 - t_1) F(t_1, t_2, t_3) \tag{26}$$

is a Massau polynomial.

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MATHEMATICS

THE THEOREM ON THE REPRESENTATION OF POSITIVELY DEFINITE FUNCTIONS FOR THE GENERALIZED OPERATION OF TRANSLATION

By B. LEWITAN

(Communicated by A. N. Kolmogoroff, Member of the Academy, 27. V. 1944)

D. Raikoff (1), using the theory of normed rings, has shown that S. Bochner's theorem on the representation of positively definite functions may be generalized to the case of commutative groups with an invariant measure. In the present note we shall show that Raikoff's method can be extended to cover the operations of generalized translation (2).

Let T^s be a family of operators of generalized translation*. Let $p(t)$ be a bounded continuous function on Z. In the sequel we shall assume that for $p(t)$ is defined a family of adjoint operators $\tilde{T}^s$ by the condition

$$\int \tilde{T}_t^s p(t)\, x(t)\, dm(t) = \int p(t)\, T_t^s x(t)\, dm(t), \quad x(t) \in L$$

Let $|\tilde{T}_t^s p(t)| \leqslant A_p \sup_{t \in Z} |p(t)|$ (A_p may depend on $p(t)$) and $\tilde{T}_t^s p(t)$ be a continuous function in the aggregate of the variables t, s.

Definition. A continuous function $p(t)$ is called positively definite, if it is bounded and if for any points $t_1, t_2, \ldots, t_n \in Z$ and any complex numbers $\rho_1, \rho_2, \ldots, \rho_n$ is satisfied the inequality

$$\sum_{i,j=1}^{n} \tilde{T}_{t_i}^{t_j} p(t)\, \rho_i \overline{\rho_j} \geqslant 0 \tag{1}$$

From (1) by means of an artifice due to Riesz (3) follows

$$\int\int \tilde{T}_t^s p(t)\, x(t)\, \overline{x(s)}\, dm(t)\, dm(s) \geqslant 0, \quad x(t) \in L \tag{2}$$

In the sequel we shall assume that $|p(t)| \leqslant C_1$ (C_1 from the condition V of generalized translation). The remaining cases may be reduced to the present one.

Put for every element $\zeta = \lambda e + x(t) \in R$

$$L(\zeta) = \lambda + \int p(t)\, x(t)\, dm(t)$$

The functional L is additive and homogeneous. Let us show that $L(\tilde{\zeta}) = \overline{L(\zeta)}$.

From (1) follows that $p(t)$ is an «Hermitian» function, *i. e.* $\tilde{T}_t^s p(t) = \tilde{T}_s^t \overline{p(s)}$.

* For the notions and notations cf. (3).

Further we have

$$L(\zeta)=\overline{\lambda}+\int p(s)\,dm(s)\lim_{n\to\infty}\int \tilde{T}_t^s\,\overline{x(t)}\,x_n(t)\,dm(t)=$$
$$=\overline{\lambda}+\int p(s)\,dm(s)\lim_{n\to\infty}\int \overline{x(t)}\,T_t^s(x)_n(t)\,dm(t)=$$
$$=\overline{\lambda}+\int \overline{x(t)}\,dm(t)\lim_{n\to\infty}\int \tilde{T}_t^s\,\overline{p(t)}\,x_n(s)\,dm(s)=$$
$$=\overline{\lambda}+\int \overline{p(t)}\,\overline{x(t)}\,dm(t)=\overline{L(\zeta)}$$

Here we assume that for $p(t)$ is satisfied the condition I of generalized translation.

Theorem 1. *L is a positive linear functional in the space C, and* $\|L\|=1$.

Proof. Evidently $|L(\zeta)|\leqslant\|\zeta\|$. Further, we have

$$x\bullet\overline{y}=\int \tilde{T}_t^s x(t)\left[\lim_{n\to\infty}\int \tilde{T}_u^s\,\overline{y(u)}\,x_n(u)\,dm(u)\right]dm(s)=$$
$$=\int \overline{y(u)}\,dm(u)\lim_{n\to\infty}\int \tilde{T}_t^s x(t)\,T_s^u x_n(s)\,dm(s).$$

In virtue of the formula (4) of [3] we have

$$x\bullet\overline{y}=\int \overline{y(u)}\,dm(u)\lim_{n\to\infty}\int \tilde{T}_t^s T_t^u x(t)\,x_n(s)\,dm(s)=\int T_t^u x(t)\,\overline{y(u)}\,dm(u)$$

Hence, in virtue of (2),

$$L(x\bullet\tilde{x})=\int p(t)\left[\int T_t^u x(t)\,\overline{x(u)}\,dm(u)\right]dm(t)=$$
$$=\int\int \tilde{T}_t^u p(t)\,x(t)\,\overline{x(u)}\,dm(t)\,dm(u)\geqslant 0$$

From this point onwards the proof of the theorem coincides with the proof of Raikoff [1], in which $y(g)$ is to be replaced by $x_n(t)$.

Theorem 2. *A continuous positively definite function* $p(t)$ *is representable in the form*

$$p(t)=\int_{\mathfrak{M}-M_0}\overline{\varphi(\alpha,t)}\,dF(\alpha)$$

where $F(\Delta)$ *is a totally additive function on Borel sets of the locally compact set* $\mathfrak{M}-M_0$.

The proof is literally the same as in the case of groups [1].

Let A_t be a family of commutative normal uniformly bounded operators depending on the element t of the space Z and defined in a certain Hilbert space H.

Consider the scalar product

$$h(t)=(A_t f,\ g)\quad (f,\ g\in H)$$

and assume that $h(t)$ is a continuous function of t.

Suppose that there exists such an operation of generalized translation T^s that for any $f,\ g\in H$

$$T_t^s h(t)=(A_s A_t f,\ g),\quad \tilde{T}_t^s h(t)=(\tilde{A}_s A_t f,\ g)\qquad (3)$$

Thus, for instance, if Z is an Abelian group, then it suffices to assume that the A_t form a group of unitary operators.

By a direct substitution we verify that $f(t) = (A_t f, f)$ is a positive function. Consequently

$$(A_t f, f) = \int \overline{\varphi(\alpha, t)}\, dF(\alpha) \tag{4}$$

From the formula (4) and the uniqueness of representation by a known artifice ([2]) we obtain the spectral formula

$$(A_t f, g) = \int \overline{\varphi(\alpha, t)}\, d(E_\alpha f, g) \tag{5}$$

Herein E_Δ is the decomposition of unity, *i. e.* satisfies the conditions:
1) $E_\theta = 0$ (θ being the void set and 0, the zero operator);
2) $E_{\mathfrak{M}-M_0} = E$ (E being the unit operator);
3) $E_{\Delta'} E_{\Delta''} = E_{(\Delta'\Delta'')}$;
4) $A_t E_\Delta = E_\Delta A_t$.

The formula (5) we shall apply in a different paper in proving the Plancherel theorem.

Observe that $\sup_\alpha |\varphi(\alpha, t)| = \|A_t\|$.

Hence follows the boundedness of $\varphi(\alpha, t)$ in the aggregate of the variables α, t.

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MATHEMATICS

ON THE NORMABILITY OF TOPOLOGICAL RINGS

By D. MILMAN

(Communicated by A. N. Kolmogoroff, Member of the Academy, 29. VI. 1944)

§ 1. I. Shafarevitch gave a criterion of normability of a topological field K (1). Under the norm of an element $x \in K$ he understands a function $\| x \|$ satisfying the following conditions:

a) $\| x \| > 0$ for $x \neq 0$ and $\| 0 \| = 0$,

b) $\| x \cdot y \| = \| x \| \cdot \| y \|$,

c) $\| x + y \| \leqslant \| x \| + \| y \|$,

d) the sets $\{ \| x \| < \varepsilon,\ \varepsilon < 0 \}$ form a complete system of neighbourhoods of 0.

Under the norm in a topological ring K * we shall understand a function $\| x \|$ satisfying the conditions a), c), d), whereas instead of the condition b) we demand the following one:

b′) $\| x \cdot y \| \leqslant \| x \| \cdot \| y \|$.

We shall call a set $W \subset K$ bounded, if it satisfies the condition: for every neighbourhood U of zero, it is possible to find a neighbourhood V of zero such that $WV \subset U$.

Under R we shall understand the aggregate of all those elements $x \in K$ for which $\lim_{n \to \infty} x^n = 0$.

Finally, we shall call the set $W \subset K$ convex, if for any two elements $x,\ y \in W$ it is possible to find an element $z \in W$ such that $2z = x + y$, and every such element $z \in K$ belongs to W.

Applying the method of Shafarevitch, we prove the following theorem.

Theorem 1. *If a set R is open, bounded, convex and contains an element p, possessing an inverse, then the topological ring K is normable.*

Proof. Let $p \in R$ and possesses an inverse in K. It is easily shown that for such an element p the aggregate of neighbourhoods $\{ p^n \cdot R \}$ is complete. We shall now show that the set-theoretical sum $\sum_{n=1}^{\infty} p^{-n} \cdot R$ coincides with K. In fact, for every $x \in K$ there is an n, for which $p^n \cdot x \in R$, and consequently $x \in p^{-n} \cdot R$. Observing that the sets $p^n \cdot R$, $n = 0, \pm 1, \pm 2, \ldots$, enclose each other, decreasing as n increases, we conclude that it is possible to introduce the notion of the index of the element $x \in K$.

---

* Under a topological ring we understand an aggregate K of elements satisfying the following conditions; α) K is an algebraic, commutative ring with a unity; β) K is a topological space; γ) the sets $x + U$ form a complete system of neighbourhoods of x, if U runs through a complete system of neighbourhoods of 0; δ) if $a \in K$ and U is a neighbourhood of 0, then it is possible to find a neighbourhood U of 0 such that $aV \subset U$; ε) if $p \in K$ possesses an inverse element and U is a neighbourhood of 0, then pU is a neighbourhood of 0.

We shall call the function $n(x)$ equal to the greatest n for which $x \in p^n \cdot R$, the index of the element x; evidently $-\infty < n(x) < +\infty$.

From the convexity of R follows the inequality

$$n\left(\frac{x+z}{2}\right) \geqslant \operatorname{Min}[n(x),\ n(y)] \tag{1}$$

Since $R \cdot R \subseteq R$,

$$n(x \cdot y) \geqslant n(x) + n(y). \tag{2}$$

Taking into account that if the power of an element belongs to R, then the element itself belongs to R, and using the inequality (2), we obtain

$$(n+1)\,k \geqslant n(x^k) \tag{3}$$

where $n = n(x)$ and $k = 1, 2, 3, \ldots$

From the inequalities (2) and (3) follows

$$1 \geqslant \alpha_k(x) - n(x) \geqslant 0, \quad k = 1, 2, 3, \ldots \tag{4}$$

where $\alpha_k(x) = \frac{n(x^k)}{k}$, $k = 1, 2, 3, \ldots$

From the inequality (4) follows the existence of the limit of the sequence $\{\alpha_k(x)\}$. Let $\lim\limits_{k\to\infty} \alpha_k(x) = \alpha(x)$.

The function $\rho^{\alpha(x)}$ for $0 < \rho < 1$ satisfies the conditions a), b'), d) and also the following condition:

c') $\rho^{\alpha\left(\frac{x+y}{2}\right)} \leqslant [\rho^{\alpha(x)} + \rho^{\alpha(y)}] \cdot \rho^{-1}$.

The inequality c') follows from the inequalities (1) and (4). Starting from the inequality

$$0 \leqslant \alpha(x) - n(x) \leqslant 1 \tag{5}$$

we may easily verify the condition d). The conditions a) and b') are clearly satisfied.

The condition c) is not satisfied by the function $\rho^{\alpha(x)}$, but we easily verify on ground of c') that

$$\rho^{\alpha(x+y)} \leqslant \rho^{c}[\rho^{\alpha(x)} + \rho^{\alpha(y)}] \tag{6}$$

where $c = \alpha(2) - 1$.

The function $\|x\| = \rho_1^{\alpha(x)}$, where

$$\rho_1 = \rho\,\frac{1}{1+a_\rho}, \quad a_\rho = \sup \frac{\rho^{\alpha(x+y)}}{\rho^{\alpha(x)} + \rho^{\alpha(y)}}$$

satisfies all the conditions a), b'), c), d) of the norm in a topological ring. The theorem is thus proved.

Starting from the condition d) for the function $\rho^{\alpha(x)}$ and the inequality (5), we arrive at the following theorem.

Theorem 2. *The function $\rho^{n(x)}$ has the same order of increase (with respect to x) as any norm determining the topology of the ring.*

Corollary. The element p in the definition of $n(x)$ may be replaced by a linear operator P in K possessing an inverse and satisfying the condition:

$$\lim_{n\to\infty} P^n x = \theta \quad \text{for all} \quad x \in K$$

We add two instances of application of the theorems stated.

1. We consider the topology of the ring as determined by the norm given by the function $\operatorname{Max}\limits_{M\in\mathfrak{M}_0} |x(M)|$, where $\mathfrak{M}_0$ is a closed subset of the set $\mathfrak{M}$ of all maximal ideals of the complete normed ring K (2) such that $x(M) = 0$ for all $M \in \mathfrak{M}_0$ implies $x = \theta$.

2. The ring of integral functions with the usual multiplication and the topology determined by the system of neighbourhoods of zero $\{M(r; f) < \varepsilon\}$, where $M(r; f) = \operatorname{Max}_{|z| \leqslant r} f(z)$. Here to every r corresponds a different topology.

§ 2. Retaining the notations already used, we shall assume that in K holds the axiom: if U is a neighbourhood of 0, then it is possible to find a neighbourhood V of 0 such that $V + V \subset U$. As a metrical space K is, in general, not complete. In the usual manner K may be completed by introducing the space of classes $\bar{K}$ of fundamental sequences. If the sequence $\{x_m\}$ is a representative of the class $\bar{x} \in \bar{K}$, then we define $\|\bar{x}\|$ by the equality $\|\bar{x}\| = \lim_{m \to \infty} \|x_m\|$.

It is easily verified that after the natural introduction of the notions «sum» and «product» of the classes*, $\bar{K}$ becomes a normed complete ring, of which K is a dense subring.

In particular, it is of interest to consider the case where $\bar{K}$ contains the field of complex (or real) numbers; hereby $\bar{K}$ is a normed ring in the sense of Gelfand ([2]).

It may be easily seen that under the norm in K, which we introduced in § 1, we shall have: $\|x^m\| = \|x\|^m$, $m = 1, 2, 3, \ldots$.

Hence follows that

$$\operatorname{Max}_{M \in \mathfrak{M}} |x(M)| = \|x\|$$

where $\mathfrak{M}$ is the set of maximal ideals in K (if $\bar{K}$ is a Gelfand ring).

Thus, $\bar{K}$ may be considered as a complete ring of functions continuous on the bicompact $\mathfrak{M}$.

On the other hand, if $\mathfrak{M}$ is a bicompact and K is a dense subring of the complete ring of functions continuous on $\mathfrak{M}$, then the set $\{\operatorname{Max}_{M \in \mathfrak{M}} |x(M)| < 1, x \in K\}$ is a set R in K, if K is considered as a topological ring with the topology given by the neighbourhoods $\{\operatorname{Max}_{M \in \mathfrak{M}} |x(M)| < \varepsilon\}$ of 0. We thus obtain:

Theorem 3. *The class L of topological rings K containing the field of rational complex (or real) numbers, in which the sets R (whose elements satisfy the condition $\lim_{m \to \infty} x^m = 0$) are open, convex and bounded, coincides with the class L_1 of dense subrings K_1 of complete linear rings of functions continuous on a certain bicompact $\mathfrak{M}$, if the neighbourhoods of 0 in K_1 have the form: $\{\operatorname{Max}_{M \in \mathfrak{M}} |x(M)| < \varepsilon, x \in K_1\}$.*

§ 3. Denote by $\tilde{R}$ the aggregate of all elements of a topological ring K satisfying the condition $e \in \overline{xR}$ (cf. ([1])).

The following theorem holds:

Theorem 4. *If K contains a set of complex (or real) numbers, $\tilde{R}$ is bounded and R is open and convex, then K is the field of complex (or real) numbers.*

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* If the fundamental sequences $\{x_m\}$ and $\{y_m\}$ are representatives of the classes $\bar{x}$ and $\bar{y}$, then $\{x_m + y_m\}$ and $\{x_m \cdot y_m\}$ are fundamental sequences and we take them as representatives of the classes $\bar{x} + \bar{y}$ and $\bar{x} \cdot \bar{y}$.

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HYDRODYNAMICS

A CONTRIBUTION TO THE PHENOMENOLOGICAL THEORY OF TURBULENCE

By V. G. NEVZGLIADOV

(Communicated by V. I. Smirnoff, Member of the Academy, 25. I. 1945)

§ 1. The equations for averaged values describing the turbulent motion of an incompressible viscous field, which were first obtained by O. Reynolds [1] by averaging the exact equations of hydrodynamics, may be written in a compact and convenient form as follows:

$$\left.\begin{aligned} \rho\frac{\overrightarrow{Dc}}{Dt} &= \rho\vec{F} - \Delta(\bar{p}+\Pi) + k\Delta\overrightarrow{\bar{c}} - \frac{\partial\Pi'_{ik}}{\partial x_k} \\ \operatorname{div}\overrightarrow{\bar{c}} &= 0 \\ \frac{3}{2}\frac{D\Pi}{Dt} &= k\Psi - \operatorname{div}\vec{P} - \Pi'_{ik}\frac{\partial\bar{u}_k}{\partial x_i} - \operatorname{div}\vec{I} \end{aligned}\right\} \qquad (1)$$

where the following notations are introduced:

$$\left.\begin{aligned} &\Pi \equiv \frac{1}{3}\rho\overline{c'^2}, \quad \vec{I} \equiv \frac{1}{2}\rho\overrightarrow{c'c'^2}, \quad \Pi'_{ik} \equiv \rho\overline{u'_k u'_k} - \Pi\delta_{ik}, \\ &\vec{P} \equiv \overrightarrow{\bar{c}'p}, \quad \Psi \equiv \left(\overrightarrow{\bar{c}'}, \overrightarrow{\Delta\bar{c}'}\right) \quad \frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \left(\overrightarrow{\bar{c}}, \Delta\right) \end{aligned}\right\} \qquad (2)$$

$u'_i \equiv u_i - \bar{u}_i$ being the components of the relative velocity $\bar{c}'$ (the velocity of turbulent pulsations); Π, the turbulent pressure equal to $^2/_3$ of the mean kinetic energy of the turbulent pulsations (an analog of temperature); $\vec{I}$, the current density of the kinetic energy of turbulent pulsations (an analog of the heat current); $-\operatorname{div}\vec{P}$, the additional mean work performed by the pressure p in unit time due to turbulent pulsations beyond the boundaries of an elementary volume moving with the averaged velocity $\overrightarrow{\bar{c}}$; $k\Psi$, the excess of the kinetic energy of turbulent pulsations dissipated into heat (in unit time within the volume indicated) over the mean work of the surface, friction forces (due to usual viscosity) performed by turbulent pulsations at the boundary of the elementary volume.

Under averaging (denoted by a bar above the letter) two operations are to be understood: first, the arithmetical mean over a small domain whose linear dimensions are determined by the characteristic length $\bar{l}$, the mean length of the

«mixing path» after Prandtl[2]; and, secondly, the arithmetical mean in time over the interval τ, the minimum time of averaging, after which the variations of the averaged quantity assume a regular character (cease to be random variables). This definition of averaging implies the same elementary volume and the same elementary interval τ for the whole current, equal respectively to the greatest values of $\overline{l^3}$ and τ occurring in the current (the smallest values are zero, if we assume a laminar boundary layer). But this is no good approximation for all turbulent currents. Those currents for which it holds good will be called currents of «one-scale turbulence». Thereby we leave out of consideration the atmospheric turbulence, for which the existence of many $\overline{l}$ (many «scales») of different order of magnitude is characteristic. Our discussion of the subject will be confined to bounded currents (for instance, currents in tubes). The five equations of system (1) will be described as the fundamental equations of the phenomenological theory of one-scale turbulence of an incompressible viscous fluid. It may be remarked that almost in every phenomenological theory the number of functions in the fundamental equations exceeds the number of the equations themselves—a circumstance characteristic of such theories. The principal object of the theory is to supplement the fundamental equations so that the problem of determination of the functions entering into them may have a definite solution.

§ 2. These supplementary equations must not necessarily be differential equations. We consider it as an experimental fact—and raise it to the rank of a physical principle—that the number of independent boundary conditions is equal to the number of fundamental equations. From this principle follows the existence of five (according to the number of fundamental equations) fundamental independent quantities. We assume that these quantities are: $\overline{u_i}$, p, Π, so that the remaining quantities, namely Π_{ik}', $\vec{I}$, $\vec{P}$, Ψ must be expressible through the fundamental ones. We shall call them, therefore, reducible quantities. The equations supplementary to the fundamental ones must be established by expressing the reducible quantities through the fundamental quantities. These equations we shall call the dynamic state equations. Here is a general assumption as to the mathematical nature of the state equations: the value of a reducible quantity at every point of the system is determined by the values of the fundamental quantities in the whole system, *i. e.* by the form of the functions. In other words, the values of the reducible quantities are functionals of the fundamental ones. The general form of the state equations will be, therefore, the integral form. For an extensive class of motions, however, the dependence of the values of a reducible quantity at a point (x) upon the values of the fundamental quantities at points (ξ), speedily decreases with the increase of the distance $(\xi - x)$. Hence in the first approximation the integral state equations may be replaced by differential equations. If we confine ourselves to the values of the fundamental quantities at the point (x) and on the «first infinitely near sphere» around it, then the reducible quantities may be expressed through

$$\overline{u_i}, p, \Pi, \frac{\partial \overline{u_i}}{\partial x_k}, \frac{\partial \overline{p}}{\partial x_i}, \frac{\partial \Pi}{\partial x_i}, \quad (i, k = 1, 2, 3) \tag{3}$$

These last quantities we shall call independent quantities of first approximation. Thus, we are able to form the state equations in the first approximation. These equations will have a limited validity and we may by means of inequalities limiting the variations of the fundamental quantities in the current (or their boundary values) determine the range of their validity. The state equations in the second approximation must be formed by using the second derivatives of the fundamental quantities.

§ 3. In constructing the state equations the phenomenological theory is guided, first, by general considerations of invariance, symmetry and simplicity and, secondly, by considerations of similarity (the analysis of dimensionalities). These general considerations are not sufficient to make the results quite definite, they

have to be supplemented by appealing to the experiment. The final results of the theory also contain undetermined constants, the values of which must be found by experiment. The phenomenological theory is only applicable to the first theoretical summing up of the experimental data, their deeper and more exact evaluation is left to the statistical theory. Phenomenological considerations of the indicated type lead to the following state equations in the first approximation (formed with the quantities (3)):

$$\left.\begin{aligned} \Pi'_{ik} &= -\varepsilon_1 \Pi \left(\frac{\partial \bar{u}_i}{\partial x_k} + \frac{\partial \bar{u}_k}{\partial x_i} \right); \quad \vec{I} = -\varepsilon_2 \Pi \nabla \Pi \\ \vec{P} &= -\varepsilon_3 \Pi \nabla \bar{p}; \quad \Psi = \frac{3}{2\rho} \triangle \Pi - g_1 \Pi - g_2 \Pi^\alpha \end{aligned}\right\} \qquad (4)$$

(in the expression for Ψ the first term may be neglected). Herein the coefficients ε_1, ε_2, ε_3, g_1, g_2 are constants; they may be reduced by analysing the dimensionalities to five absolute coefficients, the numerical values of which will be th same for all similar currents, and which, generally speaking, are functions of Reynolds' number (determined by experiment). The number of coefficients may be reduced within the limits of the phenomenological theory to three.

§ 4. There is an analogy between the field of turbulent velocity pulsations and the molecularly chaotic motion. There exists a phenomenon described as «turbulent transfer», which is similar to the molecularly chaotic transfer and may be expressed by the formula

$$\vec{S} = -\varepsilon' \bar{c}' \bar{l} \nabla \bar{R} \qquad (5)$$

S being the current density of the transferred quantity R. Resorting to the notion of «emitting layer» (introduced into the kinetic theory by Clausius and Mayer) and to that of «fluid protuberance», the following interpretation can be put upon the quantities entering into (5): l is the length of the fluid protuberance (which «shoots out» of the emitting layer), as measured from the point of its emission to the point, where its «principal part dwindles away». The point of «emission» is that in which at a given moment the pulsation velocity is a maximum (local maximum, that is in comparison with the neighbouring points). l is the mixing path» according to Prandtl. It varies in length according to the velocity with which the protuberance shoots out. Its length, or the length of the protuberance, is greater, the greater the velocity. It is natural to assume here that the one is proportional to the power of the other. For the mean length $\bar{l}$ we assume the simplest dependence, the linear one: $\bar{l} \sim \bar{c}'$. Then the formula of turbulent transfer (5) may be rewritten in the form

$$\vec{S} = -\varepsilon \Pi \nabla \bar{R} \qquad (6)$$

where R is the transferred quantity averaged at the point where the protuberance shot out. The definition of $\bar{l}$ given here as the mean length of the protuberance conforms to the concept suggested by Taylor [3], who gives the following definition of an analogous quantity: the mean transfer of an «element of turbulence» from its starting level, where it was at equilibrium with its neighbourhood, to the layer, in which it is dissipated (this definition is regarded by Richardson as questionable, since among the chaos of turbulent motions there is neither initial equilibrium, nor final rest). Formula (6) is based on the definition of l as the «main part» of the length defined by Taylor, which just represents the main part of turbulent transfer. Our definition of l differs from the definition of the mixing path, as given by means of the «mean wave length», based on the wave relation $\omega\lambda = v$. The most characteristic fact in turbulence phenomena is the mixing of the masses of the fluid—a phenomenon which can in no way

be described by a wave process. Our definition is not based on the notion of wave processes. The quantities transferred by a fluid protuberance are generally dissipated in the course of its movement; this circumstance can be allowed for by the factor ε in formula (6), which will have a different numerical value for every transferred quantity. The state equations for Π'_{ik}, $\vec{I}$, $\vec{P}$ directly obtained from the kinetic equation (6) coincide with the equations (4) obtained from formal phenomenological considerations.

§ 5. The state equations (4) substituted into the system (1) yield five equations for five fundamental quantities: $\bar{u}_1$, $\bar{u}_2$, $\bar{u}_3$, $\bar{p}$, Π. Thus, we have established a complete system of equations describing the one-scale turbulence of an incompressible viscous fluid. It remains to formulate the boundary conditions. For $\bar{u}_i$, $\bar{p}$ they are the same as in the usual hydrodynamics. For Π they uniquely follow from the obvious physical condition: that the energy of turbulent pulsations should not penetrate into the solids confining the fluid. Hence I_n, the normal component of the energy current at the boundary with the solids, is equal to zero, which is equivalent to:

$$\Pi\Big|_{\text{at boundary}} = 0, \text{ or } \frac{\partial \Pi}{\partial n}\Big|_{\text{at boundary}} = 0 \qquad (7)$$

The first condition holds for a laminar boundary layer; the second, for a turbulent one.

There was another version to the present paper which contained some critical remarks on the equations proposed by A. N. Kolmogoroff [4] with which the author had become acquainted when he had already completed his work.

But it turned out later that I had been misled into criticism by the printer's failure to reproduce these equations correctly. The circumstance was fortunately revealed to me after Kolmogoroff, who is editor of this journal, had perused my paper in manuscript. He asked me to use this occasion for restoring his equations to their proper original form. They should read as follows:

$$\frac{D\bar{v}_i}{Dt} = F_i - \frac{\partial}{\partial x_i}\left(\frac{\bar{p}}{\rho} + b\right) + A\sum_j \frac{\partial}{\partial x_j}\left[\frac{b}{\omega}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right)\right] \qquad (a)$$

$$\sum \frac{\partial \bar{v}_i}{\partial x_i} = 0 \qquad (b)$$

$$\frac{D\omega}{Dt} = -\frac{7}{10}\omega^2 + A'\sum_j \frac{\partial}{\partial x_j}\left[\frac{b}{\omega}\frac{\partial \omega}{\partial x_j}\right] \qquad (c)$$

$$\frac{Db}{Dt} = -b\omega + \frac{A}{3}\frac{b}{\omega}\varepsilon + A''\sum_j \frac{\partial}{\partial x_j}\left[\frac{b}{\omega}\frac{\partial b}{\partial x_j}\right] \qquad (d)$$

where $\omega = c\sqrt{b}/L$, c is a constant, $b = \rho^{-1}\Pi$ and L is the «outer scale» of turbulence — the «mixing path».

The system of equations (a), (b), (d) of Kolmogoroff formally coincides with my system (1), (4), if in the former we put $\omega = \text{const}$. But since his equation (c) prohibits this, our systems are essentially different.

The solution of the system (1), (4) for a stationary current leads to formulae which are in agreement with the experiment. This solution will be discussed in another paper.

In conclusion I wish to express my gratitude to V. Smirnoff, Member of the Academy, for the discussion of different problems which arose in the course of this work.

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MATHEMATICAL PHYSICS

THE INTEGRAL EQUATIONS OF MANY-DIMENSIONAL ELECTRIC MODELS

By L. I. GUTENMACHER

(Communicated by N. G. Brujewitch, Member of the Academy, 16. XII. 1944)

We are here to consider some important processes taking place in the many-dimensional models described in (1).

1. Equations with the Parameter ω and Homogeneous Equations. Let the conductances A and B consist of purely reactive elements L and C, and the sources

$$\overline{E} = Ee^{i\omega t} \qquad (i = \sqrt{-1},\ \omega, \text{ the frequency})$$

Then the conductance of bipoles of the type in Fig. 1 in (1) will be determined by the expression

$$A, B = 1 : \sum_{i=1}^{n} \left[1 : \sum_{m=1}^{k_i} \frac{1}{\omega L_{im} - \frac{1}{\omega C_{im}}} \right] \tag{1}$$

The equation for potential differences at junction-points will be as follows:

$$u\{x_k\} = F\{x_k\} + \int_{(G)} \dots \int Z\{x_k, s_l, \omega\} B\{s_l, \omega\} u\{s_l\} ds_1 \dots ds_n \tag{2}$$

The nucleus of the equation depends on the frequency ω.

In the case where the connexion resistances Z are merely inductances, and the conductances B are merely capacities, *i. e.* when

$$Z\{x_k, s_l, \omega\} = \omega L\{x_k, s_l\}$$
$$B\{s_l, \omega\} = \omega C\{s_l\}$$

we find

$$u\{x_k\} = F\{x_k\} + \omega^2 \int_{(G)} \dots \int L\{x_k, s_l\} C\{s_l\} u\{s_l\} ds_1 \dots ds_n \tag{3}$$

If we interchange inductances and capacities, *i. e.* put

$$Z'\{x_k, s_l, \omega\} = \frac{1}{\omega} \frac{1}{C}\{x_k, s_l\}$$
$$B'\{s_l, \omega\} = \frac{1}{\omega} \frac{1}{L}\{s_l\}$$

then

$$u\{x_k\} = F\{x_k\} + \frac{1}{\omega^2} \int_{(G)} \cdots \int \frac{1}{C}\{x_k, s_l\} \frac{1}{L}\{s_l\} u\{s_l\} ds_1 \ldots ds_n \qquad (4)$$

For some definite values of the frequency $\omega = \omega_\nu$ the circuits fall into resonance. These values ω_ν correspond to the characteristic frequencies of the system. If there is energy in the system and no external sources $E\{s\}$, characteristic vibrations with the frequencies ω_ν may take place.

In this case $F\{x_k\} = 0$, and the potential differences at the junction-points correspond approximately to the solutions of the following homogeneous integral equations of the second kind:

$$u\{x_k\} = \omega^2 \int_{(G)} \cdots \int L\{x_k, s_l\} C\{s_l\} u\{s_l\} ds_1 \ldots ds_n \qquad (5)$$

2. Models of Purely Active Resistances. If in the elements A and B there are only active resistances, then

$$A, B = 1 : \sum \left[1 : \sum \frac{1}{R_{im}} \right] \qquad (6)$$

the nucleus of the equation becomes positive definite. No resonance phenomena can appear. The homogeneous equation possesses only the trivial solution. The sources E may be sources of a direct as well as of an alternating current.

3. The System of Two Integral Equations. If sources $E_k = E_l e^{i(\omega t + \varphi k)}$ are acting on the circuit consisting of a combination of active (R) and reactive elements (L and C), then the equations become more involved.

Applying the symbolic method based on the representation of sine-functions of the time by complex numbers, we may write the equations for potential differences in the form of a system of two integral equations of the second kind.

The conductance of the bipole B will be

$$B = 1 : \sum_{i=1}^{n} \left[1 : \sum_{m=1}^{k_i} \frac{1}{R_{im} + i\left(\omega L_{im} - \frac{1}{\omega C_{im}}\right)} \right] = B_a - iB_b \qquad (7)$$

The case presented by formula (7) differs from the case (1) in that in (1) the conductance depends only on self-inductance and capacity, and hence is represented by the real coefficient in an imaginary quantity, whereas in (7) the conductance depends also upon the resistance R and so is a complex quantity.

Let us represent the sources F, potential differences u and resistances Z in the form of complex quantities

$$\bar{F} = F_a + iF_b, \quad \bar{u} = u_a + iu_b, \quad \bar{Z} = Z_a + iZ_b$$

The equation for potential differences at the knots of the junctions assumes the following form:

$$\left.\begin{aligned} u_a\{x_k\} &= F_a\{x_k\} + \int_{(G)} \cdots \int [(Z_a B_a + Z_b B_b) u_a\{s_l\} - \\ &\quad - (Z_b B_a - Z_a B_b) u_b\{s_l\}] ds_1 \ldots ds_n \\ u_b\{x_k\} &= F_b\{x_k\} + \int_{(G)} \cdots \int [(Z_b B_a - Z_a B_b) u_a\{s_l\} + \\ &\quad + (Z_a B_a + Z_b B_b) u_b\{s_l\}] ds_1 \ldots ds_n \end{aligned}\right\} \qquad (8)$$

Resonance phenomena in this model occur with different values of the frequency. The system is, moreover, characterized by dissipation of energy, which is lost in heating the elements of the circuit.

Integro-Differential Equations

The transient phenomena in electric models arise under the action of sources E which are variable with time: $E = E(t)$, or else they appear as the result of alterations in the state of the circuit relative to some initial conditions.

In the presence of capacities C and inductances L in the circuit the energy is accumulated in the form of electric field energy $Cu^2/2$ and in the form of magnetic field energy $LI^2/2 = \Phi I/2$ (Φ being the magnetic current). On account of the heating of the conductors there is a continuous loss of energy proportional to RI^2.

The dependence between current strength and potential difference, for instance, in the bipole of Fig. 1 in ([1]), is determined by the transient conductance.

The quantity $Z\{x_{kik}, s_{ljl}\}$ is also a differential operator, namely a function of a function of the operator $p = \partial/\partial t$.

Thus, the transient processes in such circuits are described by integro-differential equations.

Consider the following example. Let

$$u\{x_{kik}\} = \left[L\{x_{kik};\ s_{ljl}\} \frac{\partial}{\partial t} + R\{x_{kik};\ s_{ljl}\} \right] I\{s_{ljl}\}$$

and

$$I\{s_{ljl}\} = \left[C\{s_{ljl}\} \frac{\partial}{\partial t} + \frac{1}{R_0}\{s_{ljl}\} \right] u\{s_{ljl}\}$$

The equation will have the form

$$u\{x_k, t\} = F\{x_k, t\} + \int \cdots \int_{(G)} \left[LC \frac{\partial^2}{\partial t^2} + \left(RC \frac{L}{R_0} \right) \frac{\partial}{\partial t} + \frac{R}{R_0} \right] \{x_k, s_l\}\, u\{s_l, t\}\, ds_1 \ldots ds_n \qquad (9)$$

In modelling the phenomena described by equations of the type (9) arbitrary initial conditions may be set up in the circuit by charging the capacities C and letting the initial currents to pass through the inductances L.

The integro-differential equations of the type (9) may be rewritten also in the form of integral equations of the second kind of mixed Fredholm-Volterra type. To this end the time has to be considered as an additional coordinate in the definition of the nucleus $K\{x_k, t;\ s_l, \tau\}$ and in forming the equation.

The idea underlying the introduction of the resistance function $Z\{x_{kik}; s_{ljl}\}$ admits of generalization so as to cover the problem of determining the current density vector in electric models and other quantities.

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MATHEMATICAL PHYSICS

THE OSCILLATIONS OF AN ISOTROPIC ELASTIC SPHERE

By G. I. PETRASHEN

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In the present paper the method of spherical vectors is applied to problems of proper and forced oscillations of an elastic sphere. The solution given is complete, whereby it compares favourably with those obtained by previous authors (1).

1. We shall represent the displacement vector U in the form of a sum

$$\mathbf{U} = \mathbf{U}_1 + \mathbf{U}_2 \tag{1}$$

where rot $\mathbf{U}_1 = 0$, div $\mathbf{U}_2 = 0$.

The displacement vector is determined from the solutions of the equation

$$(\lambda + \mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{U} + \mu\Delta\mathbf{U} = \rho\,\frac{\partial^2\mathbf{U}}{\partial t^2} \tag{2}$$

when on the surface of the sphere $r = R$ are given either the displacements

$$\mathbf{U}(R, \vartheta, \varphi, t) = \mathbf{U}_0(\vartheta, \varphi, t) \tag{3}$$

of the tensions

$$\lambda r_1\,\mathrm{div}\,\mathbf{U} + Z\mu\left(\frac{\partial\mathbf{U}}{\partial r} + \frac{1}{2}\,[\mathbf{r}, \mathrm{rot}\,\mathbf{U}]\right)\Big|_{r=R} = \mathbf{F}(\vartheta, \varphi, t) \tag{4}$$

Using (1) we may consider as initial the following wave equations:

$$\Delta\mathbf{U}_1 - a^2\,\frac{\partial^2\mathbf{U}_1}{\partial t^2} = 0, \qquad \Delta\mathbf{U}_2 - b^2\,\frac{\partial^2\mathbf{U}_2}{\partial t^2} = 0 \tag{5}$$

where $a^2 = \rho/(\lambda + 2\mu)$ and $b^2 = \rho/\mu$.

We shall seek the solutions of these equations in the form of an expansion in spherical vectors (2):

$$\left.\begin{aligned}\mathbf{U}_1 &= \sum_{\substack{l=1\\|m|\leqslant l}}^{\infty}\{\Phi^{+1}_{lm}(r,t)\,\mathbf{Y}^{+1}_{lm} + \Phi^{-1}_{lm}(r,t)\,\mathbf{Y}^{-1}_{lm}\}\\ \mathbf{U}_2 &= \sum_{\substack{l=1\\|m|\leqslant l}}^{\infty}[\varphi^0_{lm}(r,t)\,\mathbf{Y}^0_{lm} + f^{+1}_{lm}(r,t)\,\mathbf{Y}^{+1}_{lm} + f^{-1}_{lm}(r,t)\,\mathbf{Y}^{-1}_{lm})\end{aligned}\right\} \tag{6}$$

The vectors given at the boundary will be represented in a similar form

$$\mathbf{U}_0 = \sum_{\substack{l=0\\|m|\leqslant l}}^{\infty}[a^0_{lm}(t)\,\mathbf{Y}^0_{lm} + a^{+1}_{lm}(t)\,\mathbf{Y}^{+1}_{lm} + a^{-1}_{lm}(t)\,\mathbf{Y}^{-1}_{lm}] \tag{7}$$

$$\mathbf{F} = \sum_{\substack{l=0 \\ |m|\leqslant l}}^{\infty} [F^0_{lm}(t)\,\mathbf{Y}^0_{lm} + F^{+1}_{lm}(t)\,\mathbf{Y}^{+1}_{lm} + F^{-1}_{lm}(t)\,\mathbf{Y}^{-1}_{lm}] \tag{8}$$

Substituting (6) in equations (5) and in the boundary conditions (3) and (4), we come to the problem of integration of the equations for radial functions:

$$\frac{\partial^2 \Phi^\nu_{lm}}{\partial r^2} + \frac{2}{r}\frac{\partial \Phi^\nu_{lm}}{\partial r} - \frac{(l+\nu)(l+\nu+1)}{r^2}\Phi^\nu_{lm} = a^2 \frac{\partial^2 \Phi^\nu_{lm}}{\partial t^2} \tag{9}$$

$$\frac{\partial^2 f^\nu_{lm}}{\partial r^2} + \frac{2}{r}\frac{\partial f^\nu_{lm}}{\partial r} - \frac{(l+\nu)(l+\nu+1)}{r^2} f^\nu_{lm} = b^2 \frac{\partial^2 f^\nu_{lm}}{\partial t^2} \tag{10}$$

$$\frac{\partial^2 \varphi^0_{lm}}{\partial r^2} + \frac{2}{r}\frac{\partial \varphi^0_{lm}}{\partial r} - \frac{(l+\nu)(l+\nu+1)}{r^2}\varphi^0_{lm} = b^2 \frac{\partial^2 \varphi^0_{lm}}{\partial t^2} \tag{11}$$

where $\nu = \pm 1$ and the boundary conditions are

$$\varphi^0_{lm}(R,t) = a^0_{lm}(t), \quad \Phi^\nu_{lm}(R,t) + f^\nu_{lm}(R,t) = a^\nu_{lm}(t) \tag{12}$$

if the displacements on the surface are given. If the tensions are given on the surface, the boundary conditions are:

$$\left.\begin{aligned}
&\mu\left(\frac{\partial \varphi^0_{lm}}{\partial r} - \frac{\varphi^0_{lm}}{r}\right)\Big|_{r=R} = F^0_{lm}(t) \\
&\lambda\left(\frac{\partial \Phi^{+1}_{lm}}{\partial r} + \frac{l+2}{r}\Phi^{+1}_{lm}\right) - \mu\left(\frac{\partial f^{+1}_{lm}}{\partial r} + \frac{l+2}{r} f^{+1}_{lm}\right) + \\
&+ 2\mu\left(\frac{\partial \Phi^{+1}_{lm}}{\partial r} + \frac{\partial f^{+1}_{lm}}{\partial r}\right)\Big|_{r=R} = F^{+1}_{lm}(t) \\
&\lambda\left(\frac{\partial \Phi^{-1}_{lm}}{\partial r} - \frac{l-1}{r}\Phi^{-1}_{lm}\right) - \mu\left(\frac{\partial f^{-1}_{lm}}{\partial r} - \frac{l-1}{r} f^{-1}_{lm}\right) + \\
&+ 2\mu\left(\frac{\partial \Phi^{-1}_{lm}}{\partial r} + \frac{\partial f^{-1}_{lm}}{\partial r}\right)\Big|_{r=R} = F^{-1}_{lm}(t)
\end{aligned}\right\} \tag{13}$$

In virtue of the condition (1), the radial functions Φ^{+1}_{lm} and Φ^{-1}_{lm}, as well as the functions f^{+1}_{lm} and f^{-1}_{lm}, are not independent, but rather connected by the differential relations (5) and (7) of our preceding paper ([3]).

Owing to these relations it will suffice to consider the equations (9) and (10) for only one of the values of the index ν (f. i. $\nu = -1$), the second equation being a corollary of the first and of the subsidiary conditions stated above.

2. Let us determine the types of the harmonic oscillations of the sphere with a surface free of tensions. Representing the displacement in the form

$$\mathbf{U}(r,\vartheta,\varphi,t) = \mathbf{U}(r,\vartheta,\varphi)\,e^{i\nu t}$$

and inserting this expression into (2) we come to the problem of characteristic values of the equation

$$(\lambda+\mu)\,\mathrm{grad\,div}\;\mathbf{U}_n + \mu\Delta\mathbf{U}_n + \rho\nu_n^2\mathbf{U}_n = 0$$

under boundary conditions (4) where the right-hand member is zero. The characteristic vectors $\mathbf{U}_n$ of such a problem form an orthogonal system in the sphere.

For the solution of the problem by the method of spherical vectors it is necessary, first, to substitute in equations (9), (10) and (11) the expressions of the radial functions with the explicitly written harmonic time-factor; to find the solutions of the equations thus obtained, finite at the origin and satisfying the subsidiary conditions (5) and (7) in ([3]), and after this to substitute these solutions in the boundary conditions (13) with their right-hand

side equal to zero. It appears that the boundary conditions are satisfied by the (l, m)-th term of the expansion (6). Omitting the factor $e^{\pm i\nu t}$ of the radial functions as well, as the indices l and m, it is easy to verify that the radial functions can be expressed by the following formulae:

for $l=0$

$$\varphi_0^0 = f_0^{+1} = f_0^{-1} = \Phi_0^{-1} = 0; \quad \Phi_0^{+1} = c_0 \Psi_1 (k_1 r) \tag{14}$$

for $l \geqslant 1$ and any m

$$\left.\begin{aligned} f^{-1} &= (l+1)\, b \Psi_{l-1}(k_2 r); \quad \Phi^{-1} = c\, \Psi_{l-1}(k_1 r); \\ \varphi^0 &= a \Psi_l (k_2 r); \quad f^{+1} = l b \Psi_{l+1}(k_2 r); \quad \Phi^{+1} = -c \Psi_{l+1}(k_1 r) \end{aligned}\right\} \tag{15}$$

where $k_1 = \nu \sqrt{\rho/(\lambda+2\mu)}$, $k_2 = \nu \sqrt{\rho/\mu}$, and Ψ is a function determined by the equality

$$\Psi_l (x) = \sqrt{\frac{2}{\pi}}\, x^{-1/2} J_{l+\frac{1}{2}}(x) \tag{16}$$

The substitution of the radial functions (14) and (15) in the boundary conditions shows that there exist three types of oscillation, where frequencies are determined from the known equations ([1]), and the characteristic vectors of which are represented by the following formulae:

1. The radial oscillations

$$\mathbf{U}_{on} = c_n \Psi_1 (k_1^n r)\, \mathbf{r}_1 \tag{17}$$

2. The first class oscillations (following the terminology of Lamb)

$$\mathbf{U}_{lmp} = a_{lmp} \Psi_l (k_2^p r)\, \mathbf{Y}_{lm}^0 \tag{18}$$

3. The second class oscillations

$$\begin{aligned} \mathbf{U}_{lmq} = a_{lmq} \Big\{ \Big[\frac{l}{l+1} A_2 \Psi_{l+1}(k_2^q r) - C_l \Phi_{l+1}(k_1^q r) \Big]\, \mathbf{Y}_{lm}^{+1} + \\ + \left[A_l \Psi_{l-1}(k_2^q r) + C_l \Psi_{l-1}(k_1^q r) \right] \mathbf{Y}_{lm}^{-1} \Big\} \end{aligned} \tag{19}$$

where A_l and C_l are

$$A_l = k_1^q \left[2\mu \frac{l-1}{k_1^q R} \Psi_{l-1}(k_1^q R) - (\lambda+2\mu)\, \Psi_l (k_1^q R) \right]$$

$$C_l = \mu (l+1) k_2^q \left[\Psi_l (k_2^q R) - \frac{2(l-1)}{k_2^q R} \Psi_{l-1}(k_2^q R) \right]$$

and k_ν^n, k_ν^p and k_ν^q (for $\nu = 1, 2$) give the values of k_1 and k_2 after substitution of the corresponding frequencies.

The system of the characteristic vectors (17), (18) and (19) is complete.

3. The same treatment applies to the problem of proper oscillations of a fastened sphere. The three types of oscillation are found in the same way. Only for the second class of oscillations the characteristic vectors differ slightly from the preceding ones. The frequencies of the oscillations are determined from the following equations:

1. Radial oscillations

$$J_{\frac{3}{2}}(k_1^n R) = 0 \tag{20}$$

2. First class oscillations

$$J_{l+\frac{1}{2}}(k_2^p R) = 0 \tag{21}$$

3. Second class oscillations

$$(l+1)\, J_{l+\frac{3}{2}}(k_1^q R)\, J_{l-\frac{1}{2}}(k_2^q R) + l J_{l+\frac{3}{2}}(k_2^q R)\, J_{l-\frac{1}{2}}(k_1^q R) = 0 \tag{22}$$

The influences made concerning the completeness of the system remain valid.

It may be remarked that the problem of proper oscillations of a spherical layer $0 < \varepsilon \leqslant r \leqslant R$ with the boundary conditions of either type is solved without difficulty by the method of spherical vectors.

4. Let us consider the solution of equation (2) for a sphere $r \leqslant R$ with homogeneous initial conditions and with the boundary conditions (3) or (4). Besides, we shall suppose that the vectors given on the sphere satisfy the following requirements: $\mathbf{U}_0(\vartheta, \varphi, t)$ is continuous with respect to t and vanishes for $t = 0$; $\mathbf{F}(\vartheta, \varphi, t)$ is piecewise continuous («stückweise stetig») with respect to t.

As follows from the section 1, the solution of this problem by the method of spherical vectors is very like the solution of the problem considered in [3]. In order to make use of the results obtained there, it is only necessary to take account of the relations (5) and (7) in [3] connecting the radial functions Φ_{lm}^{+1} and Φ_{lm}^{-1}, as well as the functions f_{lm}^{+1} and f_{lm}^{-1}. It is easy to verify that if it is done, the radial functions of the problem take the following form for $l = 0$

$$\Phi_0^{+1} = \frac{1}{r} \int_{t-aR-ar}^{t-aR+ar} \Phi_0(\tau) \frac{\tau + aR - t}{ar} d\tau$$

and for $l \geqslant 1$ and any m

$$\left.\begin{aligned}
\varphi^0 &= \frac{1}{r} \int_{t-bR-br}^{t-bR+br} \varphi(\tau) P_l\left(\frac{\tau + bR - t}{br}\right) d\tau \\
\Phi^{+1} &= \frac{1}{r} \int_{t-aR-ar}^{t-aR+ar} \Phi(\tau) P_{l+1}\left(\frac{\tau + aR - t}{ar}\right) d\tau; \\
f^{+1} &= \frac{l}{r} \int_{t-bR-br}^{t-bR+br} f(\tau) P_{l+1}\left(\frac{\tau + bR - t}{br}\right) d\tau \\
\Phi^{-1} &= \frac{1}{r} \int_{t-aR-ar}^{t-aR+ar} \Phi(\tau) P_{l-1}\left(\frac{\tau + aR - t}{ar}\right) d\tau; \\
f^{-1} &= -\frac{l+1}{r} \int_{t-bR-br}^{t-bR+br} f(\tau) P_{l-1}\left(\frac{\tau + bR - t}{br}\right) d\tau
\end{aligned}\right\} \quad (23)$$

where the indices l and m are omitted for brevity.

These radial functions correspond to the representation of the displacement vector $\mathbf{U}$ in the class of solutions of equation (2) with possible regular strong ruptures of continuity ([3]). The obtained solution of the problem satisfies the conditions of the uniqueness theorem. If the functions of τ in (23) vanish when $\tau < 0$, the initial conditions will be satisfied.

In order to satisfy the boundary conditions (3) it is necessary to solve the following integral equations of Volterra of the second kind:
for $l = 0$

$$\Phi_0(t) + \Phi_0(t - t_a) - \frac{1}{aR} \int_{t-t_a}^{t} \Phi_0(\tau) d\tau = Ra_0^{+1}(t)$$

for $l \geqslant 1$ and any m

$$\varphi(t) - (-1)^l \varphi(t - t_b) - \frac{1}{Rb} \int_{t-t_b}^{t} \varphi(\tau) P_l'\left(\frac{\tau + bR - t}{bR}\right) d\tau = Ra^{0\prime}(t)$$

$$\Phi(t)+(-1)^l\Phi(t-t_a)+l[f(t)+(-1)^l f(t-t_b)]-\frac{1}{aR}\int_{t-t_a}^{t}\Phi(\tau)P'_{l+1}\left(\frac{\tau+aR-t}{aR}\right)d\tau-$$

$$-\frac{l}{bR}\int_{t-t_b}^{t}f(\tau)P'_{l+1}\left(\frac{\tau+bR-t}{bR}\right)d\tau=Ra^{+1}(t)$$

$$\Phi(t)+(-1)^l\Phi(t-t_a)-(l+1)[f(t)+(-1)^l f(t-t_b)]-$$

$$-\frac{1}{aR}\int_{t-t_a}^{t}\Phi(\tau)P'_{l-1}\left(\frac{\tau+aR-\tau}{aR}\right)d\tau+\frac{l+1}{bR}\int_{t-t_b}^{t}f(\tau)P'_{l-1}\left(\frac{\tau+bR-t}{bR}\right)d\tau=Ra^{-1}(t)$$

where ′ denotes the differentiation with respect to the argument of the function; and similar equations with a somewhat more complicated kernels and with the right-hand sides containing $F_{ln}^{\nu}(t)$ in the case of the boundary conditions (4).

The first two equations do not differ from those considered in [3]. But the system of two last equations is more complicated, owing to the necessity of taking into account the two constants $t_a=2aR$ and $t_b=2bR$, while using the method of successive steps. If $t<t_a$, this system reduces to the following canonical form:

$$\Phi(t)-\frac{1}{aR}\int_{0}^{t}\Phi(\tau)\left[\frac{\tau+aR-t}{aR}P'_l\left(\frac{\tau+aR-t}{aR}\right)+P_l\left(\frac{\tau+aR-t}{aR}\right)\right]d\tau-$$

$$-\frac{l(l+1)}{bR}\int_{0}^{t}f(\tau)P_l\left(\frac{\tau+bR-t}{bR}\right)d\tau=R\frac{(l+1)a^{+1}(t)+la^{-1}(t)}{2l+1}$$

$$f(t)-\frac{1}{aR}\int_{0}^{t}\Phi(\tau)P_l\left(\frac{\tau+aR-t}{aR}\right)d\tau-\frac{1}{bR}\int_{0}^{t}f(\tau)\frac{\tau+bR-t}{bR}P'_l\left(\frac{\tau+bR-t}{bR}\right)d\tau=$$

$$=R\frac{a^{+1}(t)-a^{-1}(t)}{2l+1}$$

and is solved by the usual methods. When $t>t_a$, by introducing the appropriate notations the left-hand side can be left unaltered, while on the right-hand side terms will appear containing the values of the functions $\Phi(t)$ and $f(t)$ for the preceding moments of time, characterizing the reflection of the elastic waves from the surface of the sphere. The same takes place in the case of the boundary conditions (4).

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PHYSICS

PROPER MASSES AND MAGNETIC MOMENTS OF ELEMENTARY PARTICLES AND THE WENTZEL—DIRAC λ-PROCESS

By M. MARKOV

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Certain difficulties in the theory of interaction of elementary particles with the field can be overcome with the aid of the so-called λ-process proposed by Wentzel (1) and Dirac (2).

An essential feature of the limiting λ-process is that the corresponding diverging integrals of electrodynamics vanish when this method is applied. This signifies that in the theory the proper mass of the electron enters as a universal constant and principally does not reduce to the electromagnetic field.

It is interesting to note that in the case of other elementary particles (*e. g.* protons, neutrons) p a r t i a l reduction of their self energies to the energy of interaction with the field turns out to be a direct consequence of the λ-process. This is always the case when the field quanta have a mass of their own (for example, mesons in the case of nuclear fields).

Thus, the λ-process principally admits of the following program: the «real» proper mass is the same for all elementary particles and equals that of the electron which is a universal constant and cannot be reduced to a field. All differences in the masses of elementary particles are due to the interaction with the various fields, which leads to the appearance of new terms in this constant.

Let us consider this problem quantitatively.

In the Coulomb case we have the integral

$$\int_0^\infty dk \longrightarrow \infty \tag{1}$$

The λ-process transforms this integral into the expression

$$\int_0^\infty dk \cos \lambda_0 k = 0, \qquad \lambda_0 \longrightarrow 0 \tag{2}$$

In the case of nuclear, for example, pseudo-scalar forces [interaction function (3) $H' = q\,(4\pi)^{1/2}\,\tau_\alpha\,(\bar{\sigma}\,\mathrm{grad})\,\psi_\alpha\,(z)$] after simple calculations we obtain for the self-energy

$$W = -\frac{4}{3\pi}\,g^2 \int_0^\infty \frac{k^4}{k_0^2} \cos \lambda_0 k_0\, dk \tag{3}$$

where $k_0 = \sqrt{k^2 + \mu^2}$; μ is the self-energy of the meson

$$(g\mu)^2 \sim 0.1 \tag{4}$$

Using the relation

$$k^4 = (k_0^2 - \mu^2)^2 = k_0^4 - 2k_0^2\mu^2 + \mu^4 \tag{5}$$

we rewrite (3) as

$$W = -\frac{4}{3}\frac{g^2}{\pi}\int_0^\infty k_0^3 \cos\lambda_0 k_0\, dk + 2(g\mu)^2 \frac{4}{3\pi}\int_0^\infty \cos\lambda_0 k_0 dk -$$

$$-\frac{4}{3}\frac{(g\mu)^2}{\pi}\mu^3\int_0^\infty \frac{\cos\lambda_0 k_0}{k_0^2}\, dk \tag{6}$$

The first two integrals reduce to Bessel functions $J_n(\lambda)$ with a root at the point $\lambda = 0$.

The last integral (6) is finite. From the beginning we may put $\lambda = 0$:

$$W_1 = -\frac{4}{3}(g\mu)^2\frac{\mu}{\pi}\int_0^\infty \frac{dk}{k^2+1} \tag{7}$$

$$W_1 = -\frac{2}{3}0.1\mu \tag{8}$$

which means that about 10 electron masses are added to the proper mass.

For the vector law of interaction [3]

$$H' = g(4\pi)^{1/2}(\tau_a \operatorname{curl})\,\psi^a(z)$$

After the corresponding calculations we obtain a similar expression

$$W_2 = -\frac{4}{3}(g\mu_2)^2\mu_2 \tag{9}$$

In the case of the electron-neutrino law of interaction (theory of β-disintegration in the form initially proposed by Fermi) the expression for the self-energy of the source has the form

$$W_3 = -\frac{G^2h^5}{2^4\pi^5 m^4c^3}\int_0^\infty\int_0^\infty \frac{d^3k_n d^3k_\nu}{k_n^0 + k_\nu}\,[1 - \cos(\overline{k}_n\overline{k}_\nu)]\cos\lambda_0' k_n^0 \cos\lambda_0'' k_\nu$$

Here

$$k_n^0 = \sqrt{k_n^2 + \mu^2}; \quad k_\nu = (\overline{k}_\nu),$$

i. e. we have assumed that the neutrino does not have a rest mass. λ_0' and λ_0'' are the time components of the Wentzel-Dirac vectors (limiting process).

Upon integrating with respect to the angles we obtain

$$W_3 = -\frac{G^2h^5}{\pi m^4c^3}\int_0^\infty\int_0^\infty \frac{k_n^0\, dk_n\, k_\nu^2\, dk_\nu \cos\lambda_0' k_n^0 \cos\lambda_0'' k_\nu}{k_n^0 + k_\nu} +$$

$$+\frac{G^2h^5}{\pi m^4c^3}\mu^2\int_0^\infty\int_0^\infty \frac{dk_n\, k_\nu^2\, dk_\nu \cos\lambda_0' k_n^0 \cos\lambda_0'' k_\nu}{k_n^0 + k_\nu}$$

We see that in this case, just as in the case of light, no finite term for the self-energy of the field source exists which is not equal to zero.

If we assume that the neutrino mass is not zero and for sake of simplicity* put $\mu_n = \mu_\nu$, upon integrating we obtain

$$W_4 = + \frac{\pi}{8} G^2 mc^2 \qquad (10)$$

If the constant for the pair force theory is chosen according to Marshak-Weiskopf [4] ($G \sim 10^{-2}$), the proper mass increases by

$$\sim 10^{-4}\, mc = 10^{-2}\, m_e c^2 \qquad (11)$$

i. e. by one per cent of the electron mass.

We thus see that:

1) The interaction laws usually discussed yield values for the proper mass of nuclear particles which are too small. It is more probable that with their aid one may sooner explain the difference between the proton and neutron masses than to explain the masses of nuclear particles.

2) In the case of «pseudo-scalar» and «vector» interaction the sign of the proper mass is negative.

It should be noted that in the latter cases the sign of the magnetic moments of the nuclear particles, as Jauch [3] has shown, also turns out to be incorrect. Jauch assumed that the meson has an anomalous magnetic moment which brings the result up to the observed value.

However, one may try to solve this problem by assuming a different interaction law.

Thus, the finite terms under discussion can be obtained from expressions of the type

$$\int_0^\infty \frac{k^{2n}}{k_0^{2m}}\, dk \to \int_0^\infty \frac{(k_0^2 - \mu^2)^n}{k_0^{2m}}\, dk$$

They can be obtained by the substitution $k^2 = k_0^2 - \mu^2$ and have the form

$$(-1)^n \mu^{2n} \int_0^\infty \frac{dk}{k_0^{2m}} \qquad (12)$$

The sign of the latter expression depends on n which in turn depends on the law of interaction.

Thus, as a formal example it may be noted that the interaction law in the form

$$H' = g\,(4\pi)^{1/2}\, \tau_\alpha\, (\bar{\sigma}\, \mathrm{grad})\, \psi_\alpha(z)\, \varphi(z)$$

gives a correct sign for the nuclear particle magnetic moments. Here ψ is a pseudo-scalar describing a charged meson, and φ is a scalar describing a neutral meson of spin 0.

In this case the self-energy is given by

$$W_5 = + \frac{4g^2}{(2\pi)^3} \mu^2 \int_0^\infty \frac{dk\, dl}{k_0 l_0 (k_0 + l_0)}. \qquad (13)$$

Just as in the above case of the usual pair forces (particles with spin $^1/_2$) the expression for the proper mass is also positive here.

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* Besides the nuclear forces usually discussed in the literature, «pair forces» can be suggested, the particles being an electron of mass m_e and meson with spin $^1/_2$ and mass $\sim 100\, m_e$. These forces are very peculiar and lead to a number of specific effects. A fast proton on entering the Earth's atmosphere could be slowed down by these forces and emit an electron and meson; this may turn out to be important in the production of the soft component and does not require the existence of short-life mesons.

The results of calculations (8), (9) and (10) show that the expression for the proper masses of elementary particles which are obtained by the λ-process have the form

$$\sim G^2 u \qquad (14)$$

We thus arrive at the conclusion that the observed values of the self-energy of nuclear particles can be obtained in the λ-process theory only if a stronger interaction between the field and elementary particles is introduced.

Without completely modifying the weak bond conception which explains the particular properties of the deuton and the structure of atomic nuclei in general, one may introduce repulsive forces which act at short distance. These forces can yield correct values for the proper masses of the nuclear particles and, on the other hand, easily explain the stationary states of the deuton.

A detailed analysis of the possibilities of introducing in the theory short distance repulsive forces will be given in our next paper. As a formal example, we may point to the combination of two vector fields

$$H' = g\,(4\pi)^{1/2}\,(\tau_a \operatorname{curl})\,\varphi^a(z) + f\varphi^*\psi \qquad (15)$$

where the first term is a vector field of mesons with spin 1, and the second term is the vector case of pair forces (two mesons of spin $^1/_2$). While the first term leads to attraction according to the law $1/r^3$, the second term, as is well known, gives a repulsion law $1/r^5$. The interaction can be written in a relativistically invariant form.

In the theory nuclear forces usually discussed (Möller — Rosenfeld) there is also a combination of two fields. Repulsion forces of very high powers of $1/r$ can be introduced, this being equivalent to a very sharp relativistically invariant cut-off of the attraction forces at small distances.

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PHYSICS

AN EXTENSION OF THE LIMITS OF APPLICABILITY OF THE SMALL PARAMETER METHOD

By S. M. RYTOV

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1. The method of the small parameter, developed by A. A. Andronov, A. A. Witt and others for solving non-linear oscillation problems, is based on the Poincaré theory (which yields the periodic solutions)(1) and the Liapunov theory (in which the stability of the solutions is investigated) (2), and is in the fact a modification of the perturbation method.

As a result of this circumstance, certain restrictions are imposed on the small parameter method which affect the initial (zero) approximation, as well as the convergence of the resulting series. As is well known, within the limits of these general restrictions it was possible, by applying this method, to solve a number of problems relating to autonomic and non-autonomic non-linear oscillatory systems, and chiefly those with one or two degrees of freedom. Simultaneously some important possibilities inherent in the method were brought to light. Thus, besides the theory of phenomena caused by the action on the autonomic system of an external force of frequency near to the proper frequency of the system (and which in this respect are analogous to resonance in linear systems) (3), it turned out to be possible to develop the theory of phenomena which are dissimilar to any of those in linear systems with constant parameters.

I have in view here the excitation in the system of undertones of the applied force or the so-called resonance of the n-th kind. This extension of the range of problems which can be solved by the small parameter method was noted and developed by L. I. Mandelstam and N. D. Papalexi (4).

In this paper I should like to suggest a new line of development of this method. We suggest that not only positive, but also negative orders of magnitude with respect to the small parameter (μ) be included in the problem; in other words, besides the «normal» (or «large») quantities of the order of $\mu^0 = 1$ and the «small» quantities of the order of μ, μ^2 *etc.*, we propose to introduce «very large» quantities of the order of $1/\mu$, $1/\mu^2$ *etc.* The expediency of this introduction will be illustrated below. However, some principle features of this development should be mentioned at this point.

2. Formally, introduction of quantities of the order of $1/\mu$ does not lead to anything fundamentally new in the small parameter theory. It is always possible to raise the order of the coefficients of the differential equations—in our case, for instance, by 1 ($1/\mu \to 1$, $1 \to \mu$, $\mu \to \mu^2$, *etc.*), as a result of which the equations will contatin only positive orders of smallness, which is the usual case. Moreover, this substitution (multiplication of the equations containing $1/\mu$ by μ) is really a necessity, as Poincaré's method can be applied only if μ is contained in the equations in positive powers.

However, from the physical point of view the use of «very large quantities» is not at all trivial and for the formulation of certain problems it may be quite helpful.

According to the very idea of the small parameter method, the former is a variable quantity. By saying «μ is small» we simply affirm in a terse manner that μ can always be considered to be as small as is required in order that any of the consequences of the theory be justified. In this case all parameters in the given problem will be either those which disappear when $\mu \to 0$, or which are constant, or, finally, which increase indefinitely. Introduction of these latter quantities in problems heretofore investigated did not seem to be expedient nor essential.

In most cases physical considerations concerning the properties of the system under investigation and the essence of the problem yield the relations between the orders of magnitudes. Which of these quantities should be accepted as a magnitude of the zero order (*i. e.* as being constant and independent of μ) is a matter of arbitrary agreement. However, each of these possible agreements is equivalent to a certain physical picture, to a certain state of the system at $\mu = 0$. Although these agreements are equivalent to each other from a formal point of view, they are quite different as regards their correspondence to our physical notions, and therefore as regards their euristic value.

The following examples will clarify these points.

3. Let us take two inductively coupled linear circuits. The equation for the currents will be:

$$\left.\begin{aligned} L_1 \ddot{I}_1 + \frac{1}{C_1} I_1 &= -R_1 \dot{I}_1 - M \ddot{I}_2 \\ L_2 \ddot{I}_2 + \frac{1}{C_2} I_2 &= -R_2 \dot{I}_2 - M \ddot{I}_1 \end{aligned}\right\} \qquad (1)$$

Let us assume that the coupling of the circuits is weak and also that damping of the oscillations is small. It then seems to be natural to consider the resistances R_1, R_2 and the coefficient of mutual inductance M to be small quantities of the order of μ, whereas the partial frequencies are of the order of 1:

$$L_1 C_1 \text{ and } L_2 C_2 \sim 1, \quad R_1,\ R_2 \text{ and } M \sim \mu \qquad (2)$$

At $\mu \to 0$ the system degenerates into two independent conservative oscillatory circuits.

Suppose, further, that the wave impedance of the first circuit is of the usual magnitude, while that in the second circuit is exceedingly high, which is the case, for instance, in the equivalent quartz circuit. We accordingly put $\sqrt{L_1/C_1} \sim 1$ and $\sqrt{L_2/C_2} \sim 1/\mu$, whence in virtue of $L_1 C_1 \sim 1$, $L_2 C_2 \sim 1$, it follows that

$$L_1 \sim 1, \quad 1/C_1 \sim 1, \quad L_2 \sim 1/\mu, \quad 1/C_2 \sim 1/\mu \qquad (3)$$

In accord with (2) and (3) the following notations for the partial frequencies and coefficients of coupling and damping may be introduced:

$$1/L_1 C_1 = \omega_1^2, \quad 1/L_2 C_2 = \omega_2^2, \quad R_1/L_1 = 2\mu h_1, \quad R_2/L_2 = 2\mu^2 h_2,$$
$$M/L_1 = \mu \varkappa_1, \quad M/L_2 = \mu^2 \varkappa_2 \qquad (4)$$

where ω, h and $\varkappa$ are of the order of 1. Equations (1) then assume the form:

$$\left.\begin{aligned} \ddot{I}_1 + \omega_1^2 I_1 &= -2\mu h_1 \dot{I}_1 - \mu \varkappa_1 \ddot{I}_2 \\ \ddot{I}_2 + \omega_2^2 I_2 &= -2\mu^2 h_2 \dot{I}_2 - \mu^2 \varkappa_2 \ddot{I}_1 \end{aligned}\right\} \qquad (5)$$

Thus, due to the introduction of $L_2 \sim 1/\mu$ the equations become asymmetrical not only as regards the damping coefficient, but with respect to the

coupling coefficients as well (asymmetry only with respect to the damping coefficients can be arrived at by assuming that $R_2 \sim \mu^2$ when $L_2 \sim 1$).

Evidently, by raising the order of the coefficients in equations (1) by unity we obtain the same equations (5):

$$L_1 \sim \mu, \quad 1/C_1 \sim \mu, \quad L_2 \sim 1, \quad 1/C_2 \sim 1, \quad R_1, \; R_2 \text{ and } M \sim \mu^2.$$

We do not introduce here «very large» quantities but consider that at $\mu = 0$ the system will be different: the only constant elements now will be the reactive parameters of the second circuit, while in the first circuit we have an infinite capacity and infinitesimally small inductance. As was mentioned above, the equations will remain the same, but if the first circuit is an usual one consisting of a coil and condenser, while the second is an equivalent quartz circuit, and if we are concerned only with the interaction of the given circuit with various quartzes, then all the advantages that a physically clear picture can yield will support the first of the two possibilities mentioned above, in which L_2 is considered as a quantity of the order of $1/\mu$.

4. Finally, as an example we may cite the results of an examination of a concrete problem, the formulation of which rests on considerations analogous to the foregoing ones. Notwithstanding the practical importance of frequency control in tube oscillators by the use of quartz, to my knowledge no theory of this phenomenon has been proposed as yet. However, by assuming that quartz has a «very large» inductance of the order of $1/\mu$ it is possible to develop a theory of the stabilization phenomenon based on the small parameter method. This has been carried out by the author in collaboration with A. M. Prokhorov and M. E. Jabotinski *.

In order to keep as close as possible to the example discussed in the preceding section, we shall confine ourselves here to the consideration of a special stabilization circuit — the so-called Ziehen-circuit. This case differs from the linear problem in that the first (usual) circuit is now that of a self-excited oscillator, the frequency of which differs only slightly (of the order of μ) from the frequency of the second or quartz circuit. Then instead of equations (5) we obtain:

$$\left.\begin{aligned} \ddot{I}_1 + I_1 &= -\mu\theta_1 \dot{I}_1 - \mu x_1 \ddot{I}_2 + \mu I_a - \mu\Delta I_1 \\ \ddot{I}_2 + I_2 &= -\mu^2\theta_2 \dot{I}_2 - \mu^2 x_2 \ddot{I}_1 \end{aligned}\right\} \qquad (6)$$

where we have introduced undimensional time $t' = \omega t$, $\omega = 1/\sqrt{L_2 C_2}$ is the quartz frequency, $\mu\theta_1 = R_1/L_1\omega$ and $\mu^2\theta_2 = R_2/L_2\omega$ are the decrements, $\mu x_1 = M/L_1$ and $\mu^2 x_2 = M/L_2$ are coupling coefficients, μI_a is anode current of the tube which depends on $\dot{I}_1$ through the feedback, and $\mu\Delta = \frac{\omega_1^2 - \omega^2}{\omega^2} \approx 2\frac{\omega_1 - \omega}{\omega}$ are the relative asyntony of the circuit and quartz.

Asymmetry of the orders of smallness in equations (6) which is due to the same cause ($L_2 \sim 1/\mu$) as that in the linear equations (5), permits one not only to develop a theory of frequency controlled oscillations but also to form a graphical picture of the processes which take place during stabilization. Namely, if terms of equations (6) are taken into consideration in the order of increasing powers of μ, the following picture will be obtained: in the zero approximation ($\mu = 0$) the system consists of two independent linear and conservative oscillatory systems of the same frequency. Account of first order terms leads to the case of a self-excited oscillator which is acted upon, almost in resonance (asyntony $\mu\Delta$), by a force $-\mu x_1 \ddot{I}_2$; this is the problem of forced synchronization of self-sustained systems by an external force almost in resonance with it ([3]). The source of the force is the second oscillatory system which, as before, is independent and conservative.

* Prepared to press.

Finally, when second order quantities are taken into account, damping of the quartz appears simultaneously with the action of the tube oscillator which sustains the oscillations of the quartz (force $-\mu^2 x_2 \ddot{I}_1$). Thus, the interaction is asymmetrical: in accord with the small decrement of quartz ($\sim\mu^2$) the action of the oscillator on the quartz is very small ($\sim\mu^2$), whereas the action of the quartz on the oscillator is «strong» ($\sim\mu$) and can therefore synchronize the latter near resonance.

It seems to me that this graphical interpretation which relates frequency control with forced synchronization might hold interest.

A rigorous solution of equations (6) by the small parameter method * leads to the Ziehen-phenomenon ([5,6]) which is characterized by the peculiarity that in certain ranges of asyntony $\mu\Delta$ there exists a solution of frequency n which differs from that of quartz (1) only in the order of μ^2:

$$n = 1 - \mu^2 \frac{\theta_2}{2}\left[\frac{x_1 x_2}{2\theta_2 \Delta} + \sqrt{\left(\frac{x_1 x_2}{2\theta_2 \Delta}\right)^2 - 1}\right]$$

This is the essence of frequency control as regards the asyntony: when this latter is varied by an order of μ, the frequency of the oscillations varies as μ^2 and not as μ, which is the case of uncontrolled operation or in systems consisting of the usual circuits.

Moreover, the remaining parameters of the circuit enter the frequency only as a correction of the second order of smallness. The essential point here is that in the frequency controlled operation the frequency correction of the second order does not contain any of the parameters of the tube. Thus, the dependence of the frequency on the current supply conditions is removed at least to the third order in distinction to uncontrolled circuits and oscillations, for which the slope of the tube characteristics enters n only in the second order.

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HEAT ENGINEERING

THERMAL REGIME OF SEMI-COKING, BURNING AND GASIFICATION OF PARTICLES IN A SOLID FUEL. II

By Z. F. CHUKHANOV, Corresponding Member of the Academy

The temperature distribution along the surface of a layer of small burning particles is determined by a system of equations that includes the equations of: a) heat outlet along the layer, b) heat evolution at the surface of the fuel particles, c) heat exchange by convection with «the sources in bulk».

In certain cases the exact solution and even the construction of this system of equations presents great, sometimes insuperable, difficulties; yet in other cases again which are of practical importance the problem can be solved accurately or with satisfactory approximation.

In principle, a successful solution of the question as to the mechanism of heat outlet and the dynamics of heat evolution in the oxygen zone and of heat absorption in the reduction zone opens the possibility of solving a more general problem of the gas formation(1), and the thermal regime of burning of a solid fuel, under different conditions. The fundamental conditions for the heat being carried away along the layer is the presence of a temperature gradient, which in the case of a burning layer means that heat is consumed outside the oxygen zone. The more intense the consumption of heat, the greater the value of the heat outlet.

The outlet of heat in the direction of the burning fuel fails, if the fuel is dry, to change the maximum temperature in the zone of burning, whereas the outlet of heat into the reduction zone lowers the maximum temperature. In the case of a moist fuel where part of the heat is lost on evaporation the heat outlet to the freshly supplied fuel likewise reduces the temperature in the zone of burning. It is generally assumed that increase in the burning intensity for a unit volume of solid fuel always leads to an increase in temperature. Seeing that decrease in particle size and increase of the activity of the fuel results in increased intensity of heat evolution per unit volume of the apparatus, it might be expected from what has been said here that the temperature would increase as the particles of the fuel decrease in size, and the fuel increases in activity. As it is, however, both of these factors lead under constant loss on blowing to a decrease of temperature in the burning zone, owing to increase in outlet. If the consumption of fuel in the reduction zone is absent or suffers a decrease, which means a decrease of the temperature gradient from one layer of particles to the next, then the maximum temperature in the oxygen zone is bound to rise. This is possible if there is a carbon deficiency in the reduction zone, or else, if the activity of the fuel is extremely low, either case leading to an extension of the reduction zone. The carbon deficiency in the reduction zone may occur when a dense fuel high in ash is burnt, or when the movement of the centre of burning is too slow to ensure the «con-

sumption» of carbon in the process. The latter case will be discussed in the next communication, where a number of concrete problems will be considered on the basis of our heat elimination model.

From an analysis (1) of the thermal regime in a burning layer we derived a model of heat outlet along the layer, which for certain conditions greatly simplifies the solution of that complicate problem which consists in determining and studying the temperature regime of burning. As has been pointed out, this model is valid for a certain ratio between thermal resistances of conduction and radiation, and between those of conduction and convection. Let us set out to give a quantitative estimate of the possibility to apply our model to different conditions of burning or gasification of a solid fuel, and to different cases of heat exchange, for instance, to the conditions under which a solid fuel is converted into a semi-coke state.

1. To begin with, we will consider the conditions under which the thermal resistance of particles (conduction) upon the heat exchange by convection between the layer and the gas can be neglected. The elementary quantity of heat transferred from the gas through an elementary surface dS during the interval $d\tau$ is known to be defined by the equation

$$d^2q = \lambda_t \left| \frac{\partial T_s}{\partial z} \right|_0 dS d\tau \qquad (1)$$

where $\left| \frac{\partial T_s}{\partial z} \right|_0$ is temperature gradient at the surface of the particle, and λ_t is the thermal conductivity of a particle.

Correspondingly, we can write for the quantity of heat transferred from the gas to the surface the equation

$$d^2q = \alpha (T - T_S)\, dS\, d\tau \qquad (2)$$

It will then be obvious that

$$\alpha (T - T_S) - \lambda_t \left| \frac{\partial T_S}{\partial z} \right|_0 = 0 \qquad (3)$$

To slightly simplify the problem without affecting the essence of the process, the layer may be imagined to be composed of plates whose thickness r is equal to the mean radius of a particle. Then the equation (3) can be written as follows:

$$\alpha (T - T_S) - \lambda_t \frac{\Delta T_S}{r} = 0 \qquad (4)$$

To equation (3) there corresponds one criterion

$$\frac{\alpha r}{\lambda_t} = \text{idem} \qquad (5)$$

To distinguish it from $Nu = \alpha d/\lambda_t$, we shall denote the reciprocal quantity of this criterion by the letter N:

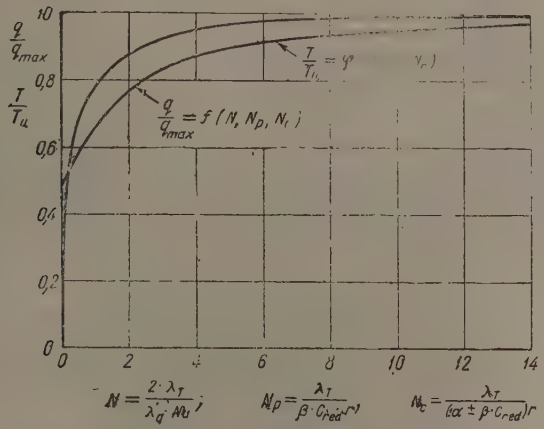
$$N = \lambda_t / \alpha r \qquad (6)$$

Obviously, the validity of (4) implies

$$N = \frac{T - T_S}{\Delta T_S} = \frac{\Delta T}{\Delta T_S} \qquad (7)$$

Thus, the dimensionless complex N gives the ratio of thermal resistances between external convection and internal conduction (along the particles) and

may serve as the criterion of applicability of our heat-outlet model in a burning layer when heat exchange by convection is allowed for. The adjoined figure represents the relation between the delivery of heat by spherical particles in the layer and the criterion N, when the temperature at the surface of the particle is half its initial temperature. On the abscissa axis the criterion N (the criterion of high-speed heat exchange), N_p and N_c (see below) are plotted, and on the ordinate axis, the ratio of the heat eliminated from the particle under certain conditions to the maximum possible elimination of heat, when the temperature is constant throughout the volume of the particle,



and also the ratio of the corresponding surface temperature to the temperature at the centre of a cooled particle.

As is evident from the figure, with $N > 2 \div 4$ the process of heat exchange in the layer will be an auto-model one with respect to N, and accordingly the total heat resistance for $N > 4$ will be defined more and more accurately by the thermal resistance of convection.

For practical determination of N the equation (6) may be transformed as follows:

$$N = \frac{2\lambda_t}{\lambda_g} \frac{1}{Nu} = 2\frac{\lambda_t}{\lambda_g} f(Re) \qquad (8)$$

A second criterion characterizing the outflow of heat with the gas and from particle to particle along the layer is the well-known criterion of Saunders ([2]):

$$Sa = \frac{wrc_p}{\lambda_g} \qquad (9)$$

The heating of the layer is automodel with respect to Sa for $Sa < 2$.

2. Representing the rows of particles in the layer as a system of screens ([1]), the flow of heat by radiation can be equated to the flow of heat by conduction in the following form:

$$\lambda_t \frac{\Delta T_S}{r} = c_{red.} \left\{ \left(\frac{T_1}{100} \right)^4 - \left(\frac{T_S}{100} \right)^4 \right\} \qquad (10)$$

or, upon transformation,

$$\lambda_t \frac{\Delta T_S}{r} = c_{red.} \beta \Delta T_1 \qquad (10a)$$

where $\beta = \frac{T_1^3 + T_1^2 T_S + T_1 T_S^2 + T_S^3}{10^8}$ depends upon the absolute values of T_1 and T_S.

By analogy with N the dimensionless complex N_p can be defined as

$$N_p = \frac{\Delta T_1}{\Delta T_S} = \frac{\lambda_t}{c_{\text{red.}} \beta r} \qquad (11)$$

The complex N_p characterizes the ratio between the temperature drop from one row of particles to the next and the difference of temperature within the solid particles themselves. The conditions for the process to be automodel with respect to the complex N_p must be determined more precisely by experiment.

However, in the absence of heat exchange by conduction the thermal resistance of the particle will, as shown by the curve in the figure, have practically no essential influence on the total thermal resistance, even if $N_p > 2 \div 4$ (just as in the case of the first criterion).

3. Let us now consider under what conditions, when the particles of the layer are heated both by convection and radiation, can we neglect the influence on the temperature field of that portion of the heat flow to the particle which is due to convection, and under what conditions again the inflow of heat by radiation can be neglected:

$$N_i = \frac{\Delta T_1}{\Delta T} = \frac{\alpha}{c_{\text{red}} \beta} = \frac{\alpha}{\alpha_p} = \frac{\lambda_g \text{Nu}}{c_{\text{red.}} \beta d} \qquad (12)$$

The increase of N_i will always mean that the rôle of the inflow of heat by convection in the total flow of heat to the particle has been increased, provided that the convection and radiation flows are of the same sign.

As will be shown later, the appraisal of the complex N_i will offer an explanation of a number of facts observed in practice, and will facilitate the analysis of the thermal regime of burning and gasification under different conditions. The heating of the layer by conduction is necessarily connected with all kinds of heat exchange, and it is only the radiation by the gas that can be safely left out of consideration under ordinary conditions and in the case of particles of small size, even within a wide range of temperature variation. If, however, the velocities of the gas are very small and the concentrations of CO_2 and H_2O in the gas are considerable, even the radiation by the gas may appreciably influence the heat exchange within the layer.

It is apparent from what has been said here that the influence of this or that kind of heat exchange upon the total heat exchange under conditions where the layer is penetrated by hot flow of gas needs to be taken into account.

On analysing the complex N_i one sees that with low temperatures, ranging 200—300° C, the heat exchange by radiation within the layer cannot appreciably affect the total heat exchange unless $\alpha < 20$—30 cal/m² hour °C, which is a very insignificant magnitude for the heat exchange by conduction. From (12) it follows that for given values of T_1 and T_2 the quantity N_i will depend either on α or on the value of Nu*. Even if the temperature attains a value not above 1000° C. the rôle of heat exchange by radiation in the layer is sharply changed, because the value of $c_{\text{red}}\beta$ in this case exceeds 200 cal/m² hour °C, and so cannot merely influence the heat exchange, but practically determines it in some cases.

The above factors controlling the heat exchange within the layer will not become conspicuous unless the thermal conductivity of particles or, more accurately, of the thermal resistance of the solid particles of the layer, has practically no influence.

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* The value of Nu for the heat exchange between the gas and the layer of solid particles in the region of $Re > 40$ can be determined with an accuracy sufficient for the practice by the following empirical equation: $Nu = 0.24 \cdot Re^{0.83}$.

Comparison of the respective criteria shows that the conditions for the elimination of heat conduction are not the same in the case of the convection and radiation types of heat exchange.

If the inflow of heat by convection and radiation to the surface of solid particles plays a considerable or, at any rate, a practically significant rôle, the applicability of our model of heat exchange, or, in other words, the condition for practical equality of temperature in the particles throughout the volume, must be determined by means of the criterion:

$$N_c = \frac{\lambda_t}{(\alpha \pm \beta c_{red.})r}$$

The size of the solid particles which permits one to neglect, with practically sufficient approximation, their thermal conductivity in computing the heat exchange in the layer, depends upon the thermal conductivity of the material they are made up, and upon the given conditions of heat exchange. Though in the case of a solid fuel this size under usual conditions is confined to a few millimetres (10—15), it nevertheless is measured in hundredths and even tenths of a metre in the case of metallic particles. It follows also from equation (8) that for particles of a given material and for a given fluid the thermal conductivity of particles in the case of heat exchange by convection can be neglected when the values of $Nu_{red.}$ rank sufficiently low:

$$Nu_{red.} \approx 0.5 \frac{\lambda_t}{\lambda_g} \qquad (13)$$

The possibilities of intensifying the process of heat exchange in the layer are directly connected with the problem discussed here. If we succeed, upon the exchange of heat, to eliminate the influence of the thermal resistance or the thermal conductivity of the particles (the walls of the heat-exchange apparatus), then we will be able to use to a full measure the exceptionally favourable hydrodynamic condition of the heat exchange by convection (which is a high-speed type of heat exchange) in a layer of spherical or irregularly shaped solid particles. In the case of burning and gasification of fuels this circumstance makes it possible under certain conditions to use the heat exchange by radiation between the burning particles for controlling the thermal regime.

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HYDROLOGY

BED LOAD MOVEMENT

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To give a picture of bed load movement, conformable to the present views on the subject, one may describe this process as follows. Of the number of particles lying at the bottom some are lifted under the influence of favourable kinematic conditions and transferred some distance away—how far away depends partly upon the given combination of instantaneous velocities. Then they fall to the bottom again to wait for the advent of another favourable combination of velocities which will make them move farther. Now, the distribution of instantaneous velocities in a turbulent flow as well as the form and size of the individual solid particles is of a decidedly random character. Therefore, the laws of their movement can only be expressed statistically. The first attempt to work out a theory of this kind, based on statistical analysis, was made by Einstein (1). But he failed by this means to come directly to the relation sought for, which in another work of that author is obtained in a purely empirical way (2).

In the present paper the problem is solved by a different method. The reasoning from beginning to end is entirely based on the statistics of turbulent fluctuations of velocity, and experimental data are only used at some links of the whole structure, whenever it is necessary to evaluate the parameters or to establish their empirical relations to one another, but this without affecting the theoretical frame of the dependence of the transportation rate upon the size of the particles and the velocity of the flow. This dependence is derived from a set of generally recognized and hardly questionable statements of the modern theory of turbulence.

Statistically, however, the ultimate relation rests on a proposition which, even though it may be only an approximation, appears to be true beyond doubt. Namely, it is assumed that the continuous process which consists in the quantity x receiving without intermission random increments according to the normal distribution law, can be divided with respect to time into such intervals that the correlation between the average value of x for one interval and its average value for the next interval will be close to zero.

Let us now proceed to derive the relation which it is the object of this work to establish. We shall confine ourselves to the case of a plane uniform flow, because this simplest case has alone been analysed in the modern theories of turbulent flow.

The distribution of the particles in the bed load is supposed to be stationary and their mechanical composition uniform. We shall use the following notation throughout this paper: p, rate of transportation expressed as volume of sediments per unit breadth of the flow; D, mean diameter of the transported particles; W, velocity of transfer by the flow of the lifted particles; u (resp. v), the value of the longitudinal (transverse) component of the instantaneous velo-

city above the motionless layer of bed load; w, the settling velocity of the particles in stagnant water.

The totality of suspended particles crossing a given section * in unit time forms the discharge of solid matter

$$p = NAD^3 \tag{1}$$

where A is a coefficient of form.

In order to simplify the statistical computations, the variation of velocity components, which is a process continuous in time, will be transformed, as has been pointed out above, into an intermittent process with its step length T not shorter than the time-interval within which vanishes the correlation between two successive velocities referring to different instants

$$R(u_t,\ u_{t+T}) = 0;\ R(v_t, v_{t+T}) = 0$$

Correspondingly, the path travelled by the flow will be also divided into intervals equal to $L = TW$.

The values of the velocity components u and v, which are now averages over time-intervals T, satisfy, as is usually assumed, to the normal distribution of Gauss.

Let us now introduce two probabilities.

1. The probability η that an arbitrary particle lying at the bottom will be lifted in course of the interval T and hence carried away, or, in other words, that the hydraulic lift will be not less than the proper weight of the particle

$$S \geqslant P$$

2. The probability ε that a lifted particle will not fall to the bottom until the end of the interval T, or, in other words, that it will take a step of length L.

Let us compute these probabilities. The hydraulic lift on the particle is $B\rho \cdot D^2u^2$, where B is a coefficient depending on the form of the particle. The weight of the particle under water is $A(\rho_S - \rho)gD^3$. The inequality $S \geqslant P$ will now take the form

$$Bu^2 \geqslant AagD \qquad \left(a = \frac{\rho_S}{\rho} - 1\right)$$

or

$$u \geqslant C\sqrt{gD} \qquad \left(C^2 = \frac{Aa}{B}\right)$$

Seeing that the longitudinal component of the instantaneous velocity satisfies Gauss' law, we have the following expression for the probability η:

$$\eta = \frac{1}{\sqrt{2\pi}\sigma_u} \int_{C\sqrt{gD}}^{\infty} e^{-\frac{(u-\bar{u})^2}{2\sigma_u^2}}\, du$$

or after the substitution

$$\frac{u-\bar{u}}{\sigma_u} = z;\ \frac{C\sqrt{gD} - \bar{u}}{\sigma_u} = \mu$$

we obtain ultimately

$$\eta = \frac{1}{2} - \Phi(\mu) \tag{2}$$

where use has been made of the customary symbol

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{z^2}{2}}\, dz$$

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* Pure rolling, which occurs rather seldom, enters into the scheme as the limiting case when the lifting height of particles is vanishing.

In a similar way is computed the probability ε that the vertical component of the velocity will be exceeding the velocity of the downward fall due to gravity during the interval T:

$$\varepsilon = \frac{1}{\sqrt{2\pi}\sigma_v} \int_w^\infty e^{-\frac{v^2}{2\sigma_v^2}} dv$$

after the substitution

$$v/\sigma_v = z; \quad w/\sigma_v = \nu$$

we have

$$\varepsilon = \frac{1}{2} - \Phi(\nu) \tag{3}$$

Now we can compute the number N in equation (1). We mark off the segments of length L upstream beginning from the computed section. Number N is the total number of particles carried down to the computed section from the first, second, third, *etc.*, segment. For the first segment this number is evidently $\frac{L}{D^2}\eta\varepsilon$.

Using further, on the basis of our proposition stated earlier, the theorem on multiplication of probabilities, we obtain for the second segment $\frac{L}{D^2}\eta\varepsilon^2$, and for the third $\frac{L}{D^2}\eta\varepsilon^3$. Summing up to infinity, we shall have

$$N = \frac{L}{D^2}\eta\{\varepsilon + \varepsilon^2 + \ldots\} = \frac{L}{D^2}\frac{\eta\varepsilon}{1-\varepsilon}$$

Substituting in (1) this expression and disclosing the values of η and ε we finally get

$$p = ADW\left[\frac{\left\{\frac{1}{2} - \Phi(\mu)\right\}\left\{\frac{1}{2} - \Phi(\nu)\right\}}{\frac{1}{2} + \Phi(\nu)}\right] \tag{4}$$

The first factors are the product of the height D and the velocity W multiplied by empirical coefficient A; this represents the fictitious rate of dense motion. The factor in brackets (which is apparently always below unity) expresses that portion of the dense motion which corresponds to some given values of D, W, $\bar{u}$, σ_u, σ_v that determine the parameters μ and ν. If D and w are large, for instance, or $\bar{u}$ is small, the functions $\Phi(\mu)$ and $\Phi(\nu)$ both tend to $\frac{1}{2}$ as a limit. Accordingly, $p \to 0$. On the other hand, with D and w decreasing or $\bar{u}$ increasing we have

$$\Phi(\mu) \to -\frac{1}{2}; \quad \Phi(\nu) \to 0$$

consequently, p tends to its conceivable limit

$$p \to ADW$$

Let us point out in conclusion that between D and w there is a definite relation which is reliably representable by empirical formulae; and that the velocity of individual particles W can with good reason and satisfactory approximation be equated to the longitudinal velocity $\bar{u}$. In computing σ_u and σ_v for a given value of $\bar{u}$ use is to be made of the experimental data available in the literature.

Consequently, our theoretical formula (4) can be easily given a form convenient for practical calculations.

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CHEMISTRY

THE STRUCTURE AND PROPERTIES OF THE MONOVALENT PEROXIDE GROUP

By V. KASSATOCHKIN

(Communicated by V. G. Chlopin, Member of the Academy, 18. VII. 1944)

Experimental evidence accumulated within recent years testifies to the fact that the monovalent peroxide group plays a definite rôle in the chemistry of many important processes requiring molecular oxygen.

Thus, in the chain reaction of hydrogen oxidation by molecular oxygen the HO_2 radical presumably takes part in the development of the chains ([1],[2]). Obviously, HO_2 radicals participate also in the catalytic decomposition of hydrogen peroxide ([3]), in the oxidation of bivalent iron in solutions([4]), *etc.*, playing the rôle of active centres of chain development.

A similar rôle in the gaseous chain reactions of autooxidation of hydrocarbons is played by peroxide RO_2 radicals ([5]).

In all the above reactions, the life-time of the monovalent peroxide group fixed in the radicals (and constituting an intermediate stage in the chain process) is small.

This accounts for the difficulties involved in the experimental study of the group. Yet, there exists an interesting case of fixation of the monovalent peroxide group in the crystalline lattice of the higher peroxides of alkaline metals (KO_2, RbO_2, CsO_2) in the form of the monovalent molecular O_2'-ion).

The crystalline lattice of the higher potassium peroxide studied by the author roentgenographically ([7]) is a simple structure—a somewhat deformed type of potassium chloride lattice with alternating potassium ions and monovalent ions of the oxygen molecule ([6],[7]). The size of the monovalent O_2'-ion can thus be accurately computed to be compared with that of the neutral oxygen molecule and of the common bivalent peroxide O_2''-ion (table).

	d_{O-O} Å	l_{min} Å	l_{maj} Å	$\frac{l_{min}}{l_{maj}}$	φ Å^3	Electronic structure	Energy of chain rupture, eV
O_2	1.20	2.40	3.60	0.67	10.6	$O \overset{\cdots}{\underset{\cdots}{=}} O$	5.03
O_2'	1.27	3.04	4.05	0.75	19.5	$[O \overset{\cdots}{\underset{\cdots}{=}} O]'$	3.82
O_2''	1.35	3.34	5.15	0.65	29.9	$[O - O]''$	2.00

The value of the minor diameter (l_{min}) of the oxygen molecule, whose shape is assumed to be that of an ellipsoid of revolution, may be determined from the size of the crystalline lattice of the γ-modification of solid oxygen.

According to Vegard, as well as Keesom and Takonis, this modification has the structure of a dense sphere packing formed by two rotating O_4-groups ([8],[9]).

If we proceed on the assumption that the O_4-group is formed by two tangential ellipsoidal O_2 molecules having their major axes most probably parallel to each other, then the size of the minor diameter will be found to equal the radius of the rotating O_4-group. Hence $l_{min}=a\sqrt{2}/4=2.40$Å, where $a=6.83$ Å, the constant of the face-centered cubic lattice.

The major diameter of the molecule equals the tripled interatomic distance $l_{maj}=3d_{O-O}=3.60$ Å, if the oxygen molecule is imagined to be composed of two spherical atoms whose centres are separated by a distance $d_{O-O}=1.20$ Å ([10]) (according to roentgenographic and spectroscopic data), and the radius of each atom is half the minor diameter of the molecule. A value close to 3.60 Å is also derived from the viscosity coefficient of oxygen ([11]). For the sake of comparison with the monovalent O_2'-ion the size of the bivalent (O_2'') ion obtained by Bernal *et al.*([12]) by analysing the structure of the crystalline lattice of SrO_2 and BaO_2 peroxides, is to be reduced to the monovalent size ([13]).

It will appear from the table that the transition from the neutral molecule to the monovalent molecular ion, and then to the bivalent ion is accompanied by a progressive increase in size and in the distance between the centres of the oxygen atoms.

Interatomic distances are an important criterion to decide upon the character of the chemical bond.

The increase in the interatomic distances upon addition of electrons to the neutral oxygen molecule appears to be a consequence of the successive rupture of two three-electron resonance bonds contained in the molecule ([6], [13]).

$$O\overset{\cdot\cdot\cdot}{\underset{\cdot\cdot\cdot}{\cdot\cdot\cdot}}O + \varepsilon \longrightarrow [O\overset{\cdot\cdot}{\underset{\cdot\cdot}{\cdot\cdot}}O]' + \varepsilon \longrightarrow [O-O]''$$

The size of the monovalent molecular oxygen ion is comparable with that of the spherical chloride ion ($l_{min} < 2R_{Cl'} < l_{maj}$).

Owing to structural likeness, the lattice energies of the superior potassium peroxide and of potassium chloride probably show but a little difference: $U_{KO_2} \cong U_{KCl} = 7.24$ eV. The opportunity is thus offered for an approximate estimate of the dissociation energy of the monovalent molecular $D_{O_2'}$-ion and of the electron affinity of the oxygen molecule A_{O_2} from the following formulae ([11]):

$$A_{O_2} = S_K + q_{KO_2} + I_K - U_{KO_2}; \quad D_{O_2'} = D_{O_2} + A_{O_2} - A_O$$

Here $D_{O_2}=5.09$ eV is the dissociation energy of the oxygen molecule; $A_O=2.2$ eV is the electron affinity of the oxygen atom; $I_K=4.32$ eV is the ionization potential of potassium; $S_K=0.94$ eV is the sublimation energy of the potassium; $q_{KO_2}=2.91$ eV is the energy of formation of KO_2.

The energy of dissociation of the monovalent molecular ion into the monovalent atomic ion and neutral oxygen atom is $D_{O_2'}=3.82$ eV, while the electron affinity of the oxygen molecule is $A_{O_2}=0.93$ eV. However, the accuracy of our computations depends on the difference in the energy values of the KO_2 and KCl lattices. Comparing the interatomic distances in the O_2'-ion and the electron affinity of the oxygen molecule with the relative position of the energy minimum on the potential curves for this ion at different states (as offered by Massey ([14]), one might be led to conclude that the monovalent molecular oxygen ion O_2' in the crystalline lattice of KO_2 is largely in the Π_g-state. The electronic structure of this ion is as follows:

$$(\sigma_g 2s)^2\,(\sigma_u 2s)^2\,(\sigma_g 2p)^2\,(\pi_u 2p)^4\,(\pi_g 2p)^3\,\Pi_g$$

This conclusion is at variance with Neuman's ([15]) thesis that the polymorphous transformation of KO_2 at -75°C is to be accounted for by the transition of the components of the electronic state of the ion $\Pi_u \longrightarrow \Pi_g$.

It is apparently the positive sign of electron affinity or, in other words, the effect of energy diminution upon the addition of the electron to the oxygen molecule, that may possibly account for both the chemical activity of molecular oxygen (its oxidative action) and the greater probability of formation of the monovalent peroxide group, as the primary act in the various processes of interaction of the oxygen molecule.

Thus, the totality of the numerous processes associated with the formation and decomposition of peroxides, in which the oxygen molecule takes part, as well as the oxidation-reduction properties of peroxides, can be described by a general electronic scheme of the monovalent course of the reaction

$$O\overset{\cdot\cdot}{\underset{\cdot\cdot}{::}}O + \varepsilon \rightleftarrows [O\overset{\cdot}{::}O]' + \varepsilon \rightleftarrows [O-O]'' + \varepsilon \rightleftarrows O' + O''$$

(the last stage of decomposition of peroxides and the reverse process of recombination of radicals and atomic ions may proceed otherwise).

According to the above scheme, at the first stage of chemical interaction one three-electron resonance bond is ruptured in the oxygen molecule owing to the addition of the electron of the oxidized molecule or atom. A peroxide of the type $R-[O\overset{\cdot}{::}O]$ is thereby formed which possesses the non-saturated properties of the free radical and is transformed upon further interaction into the common peroxide of the type $R-[O-O]-R$.

The transition into the common peroxide is accompanied by a rupture of the second three-electron bond.

The above may be illustrated by the ferro-ferri oxidation by molecular oxygen (4)

$$Fe^{\cdot\cdot} + O\overset{\cdot\cdot}{\underset{\cdot\cdot}{::}}O \rightarrow Fe^{\cdot\cdot\cdot} + [O\overset{\cdot}{::}O]'; \quad [O\overset{\cdot}{::}O]' + H^{\cdot} \rightleftarrows H[O\overset{\cdot}{::}O];$$

$$Fe^{\cdot\cdot} + H[O\overset{\cdot}{::}O] \rightarrow Fe^{\cdot\cdot\cdot} + H[O-O]'; \quad H[O-O]' + H^{\cdot} \rightleftarrows H[O-O]H$$

Here belongs also the catalytic decomposition of hydrogen peroxide in an acid solution under the influence of trivalent iron (3):

$$Fe^{\cdot\cdot\cdot} + H[O-O]' \rightleftarrows Fe^{\cdot\cdot} + H[O\overset{\cdot}{::}O]$$

$$Fe^{\cdot\cdot\cdot} + H[O\overset{\cdot}{::}O] \rightleftarrows Fe^{\cdot\cdot} + H^{\cdot} + O\overset{\cdot\cdot}{\underset{\cdot\cdot}{::}}O$$

The interaction of the oxygen molecule with organic substances also involves the participation of the RO_2 peroxide radical. The oxidation of pentane which, according to Ubbehlode (5), proceeds through the alkyl and peroxide radicals, conforms well to the electronic scheme:

$$C_5H_{11} + O\overset{\cdot\cdot}{\underset{\cdot\cdot}{::}}O \rightarrow C_5H_{11}[O\overset{\cdot}{::}O]; \quad C_5H_{11}[O\overset{\cdot}{::}O] + \\ + C_5H_{12} \rightarrow C_5H_{11}[O-O]H + C_5H_{11}$$

To account for the formation of aldehydes, alcohols and other products of oxidation of hydrocarbons, it is necessary to consider the last phase of the electronic scheme of decomposition of the peroxide group. New centres of chains may arise, such as aldehyde, hydroxyl, as well as HO_2 radicals.

As a particular case of the above electronic scheme one may point to an electron exchange between the bivalent peroxide group and the oxygen molecule with the formation of the intermediate monovalent peroxide group

$$O\overset{\cdot\cdot}{\underset{\cdot\cdot}{::}}O + [O-O]'' \rightleftarrows 2[O\overset{\cdot}{::}O]'$$

e. g. the formation of the superior peroxide KO_2 upon interaction of potassium peroxide with molecular oxygen:

$$O_2 + K_2O_2 \rightleftarrows 2KO_2$$

In this reaction the oxygen molecule fixes one electron of the bivalent peroxide ion. This results in the formation of two monovalent peroxide ions.

The above electronic scheme describes the possible and probable course of transformation of the oxygen molecule and may therefore be also regarded as an additional criterion for selecting the possible elementary processes in interpreting the results obtained by studying the kinetics and chemism of autoxidation reactions.

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PHYSICAL CHEMISTRY

ON THE EXPLOSIVE DECOMPOSITION OF CYCLOPENTADIENE AT HIGH PRESSURE

By L. F. VERESHCHAGIN and A. M. POLIAKOVA

(Communicated by N. D. Zelinsky, Member of the Academy, 19. VII. 1944)

The explosive decomposition of cyclopentadiene at a hydrostatic pressure of the order of several thousand atmospheres has been described in detail by Raistrik, Sapiro and Newitt (1), who established that decomposition begins only when a certain quite definite «critical» pressure has been reached on the temperature range 10—40 C. This minimum critical pressure, as the authors call it, was found by them to depend on the initial temperature of the sample under investigation and on the diameter and material of the ampule in which experiment was carried out. The decomposition products consisted of methane (92 per cent), hydrogen (8 per cent) and a carbonaceous residue.

We thought it of interest to investigate the explosive decomposition of cyclopentadiene under the influence of a wave produced by the explosion of the capsule of a detonator containing 1.1 g of tetryl. The pressure developing under these conditions is probably of the order of 100 000 atmospheres. These experiments were set up in a specially designed bomb that made it possible to measure the quantity proportional to the energy of the explosion, according to J. B. Chariton, from the deflexion of a lead membrane. It was found that not only cyclopentadiene but also cyclohexadiene-1,3 detonates in a steel ampule under the influence of a detonator. However, under these conditions decomposition of cyclopentadiene and cyclohexadiene is not complete if the samples taken exceed 5 cm^3 in volume. This may be easily explained by the rapid damping of the detonation wave as it spreads in the liquid. In the dimer of cyclopentadiene no detonation could be obtained.

We decided to reproduce Raistrik, Sapiro and Newitt's experiments on the decomposition of cyclopentadiene under static pressure, and got negative results. We presumed that the reason for this might perhaps lie in the presence of a small quantity of the dimer in a fresh specimen of cyclopentadiene prepared by fractional distillation of the monomer (the index of refraction for our product was $n_d^{25} = 1.4420$ as compared with the value $n_d^{25} = 1.4404$ given by the above authors). However, in our experiments we found that a highly important rôle in the explosive decomposition of cyclopentadiene is played by the rate at which the pressure increases. When the pressure increased at a rate of 1000 atmospheres per minute, we could not observe any changes whatsoever in the product except dimerization. Only after we had designed and built an apparatus making it possible to raise the pressure to the necessary critical value at a rate of 4000 atmospheres in 4 seconds, did we succeed in reproducing the phenomenon of the explosive decomposition of this compound at a temperature of 50°C in the reac-

tion vessel. In subsequent experiments we obtained explosive decomposition at smaller rates of increase in pressure as well (in the cited paper it is pointed out that to observe the effect of the explosive decomposition of cyclopentadiene a certain critical rate is necessary, which was found to be equal to 1500 atmospheres per minute).

It is interesting that explosive decomposition does not take place immediately after the necessary critical pressure is reached, but has an induction period of 5 seconds. On the basis of these experiments which will be continued, we think that the explosive decomposition is thermal in nature. It may be assumed that at a high pressure the rate of polymerization is so great that the heat evolved is sufficient to produce explosion.

The critical rate of rise in pressure is necessary here, evidently, to attain the temperature required for the rate of reaction to be sufficiently great. This temperature is attained in the reaction vessel upon adiabatic (or nearly adiabatic) compression of the liquid. As our experiments showed, when the pressure does not rise very rapidly, a certain quantity of the dimer has time to form in hydrocarbon which interferes with the reaction proceeding at a big rate. Here a rise in pressure of even 2—2.5 times the necessary critical value does not lead to explosive decomposition. These results are in agreement with the conclusions arrived at by Raistrik, Sapiro and Newitt in another paper [2]; there they showed that polymerization is dependent upon preliminary dissociation of dicyclopentadiene. Hence an experiment had to be set up in such a manner that a quantity of the dimer sufficient to retard the reaction did not form in the compound by the time the critical pressure was reached. The induction period is equal to the time necessary for the temperature to raise to such a value that the rate of the reaction corresponding to it will be the rate of the explosion.

The entire process as a whole is analogous evidently to the process of the explosive decomposition of acetylene observed by Berthelot and Wieille [3]. These authors studied the detonation of liquid acetylene by mercury fulminate, which leads to a powerful explosion after which the fragments of the bomb were found to be covered with carbon liberated during the decomposition of the acetylene.

We consider it our pleasant duty to tender our thanks to N. D. Zelinsky for the constant interest he has shown in this work.

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PHYSICAL CHEMISTRY

THE DETERMINATION OF THE FLOW VELOCITY OF THE DETONATION PRODUCTS OF SOME GASEOUS MIXTURES

By L. D. LANDAU and K. P. STANIUKOVICH

(Communicated by P. L. Kapitza, Member of the Academy, 24. VII. 1944)

The distribution of the detonation products of gaseous mixtures for a one-dimensional case may be directly obtained from the Riemann solution of the fundamental equations of hydrodynamics. This solution is based upon the assumption that $u = f(v)$, where v is the specific volume, and u is the velocity of the particles.

It follows then that

$$du = \pm \sqrt{-dp\,dv} \qquad (1)$$

where p is the pressure.

Integrating this expression, we get for a perfect gas:

$$u = u_{\text{initial}} + \frac{2}{\gamma - 1} [c_{\text{initial}} - c] \qquad (2)$$

where u_{initial} is the velocity of the medium behind the detonation-wave front, c_{initial} is the initial, and c, the local velocity of sound, $\gamma = c_p/c_v$.

Since

$$u_{\text{initial}} = \frac{D}{\gamma + 1} \qquad (3)$$

$$c_{\text{initial}} = \frac{\gamma D}{\gamma + 1} \qquad (3a)$$

where D is the detonation velocity, we may write

$$u = \frac{3\gamma - 1}{\gamma^2 - 1} D - \frac{2c}{\gamma - 1} \qquad (4)$$

When the gases flow into vacuum, $c = 0$; when they flow into the atmosphere, the value of c may be found from the condition:

$$c = c_{\text{initial}} \left(\frac{p}{p_{\text{initial}}} \right)^{\frac{\gamma - 1}{2\gamma}}$$

Substituting (3a) for c_{initial}, we obtain:

$$c = \frac{\gamma D}{\gamma + 1} \left(\frac{p}{p_{\text{initial}}} \right)^{\frac{\gamma - 1}{2\gamma}}$$

The value of p is determined from the condition that the pressure of the decomposition products (p_x) and that of the shock-wave moving in front of them

(p_{shock}) are equal.

Since

$$p_{shock} = \frac{\gamma_{atm}+1}{2}\,\rho_{atm}\,u^2$$

where ρ_{atm} is the density of the atmosphere, and γ_{atm} is the ratio of the specific heats, then

$$u = \frac{D}{\gamma^2-1}\left[3\gamma-1-2\gamma\,u^{\frac{\gamma-1}{\gamma}}\left(\frac{(\gamma_{atm}+1)\,\rho_{atm}}{2p_{initial}}\right)^{\frac{\gamma-1}{2\gamma}}\right] \qquad (4a)$$

The value of $p_{initial}$ may be determined by the formula:

$$p_{initial} = \frac{\rho_0}{\gamma+1}\,D^2 \qquad (5)$$

where ρ_0 is the density of the gaseous mixture.

The maximum density and temperature at the wave-front are determined by the formulae:

$$\rho_{initial} = \frac{\gamma+1}{\gamma}\,\rho_0 \qquad (6)$$

$$T_{initial} = \frac{2\gamma}{\gamma+1}\,T_2 \qquad (6a)$$

where T_2 is the reaction temperature. In the above formulae we did not take into account the change in the specific heats with temperature, and if instead of γ we use its value for some mean temperature between T_2 and $T_{initial}$, as is usually done, the results will not be quite accurate, especially in a determination of the flow velocity.

The value of γ_{atm} should correspond to the temperature at the front of the shock-wave; however, it may be assumed that $\gamma_{atm} = 1.4$. To determine the velocity of flow more accurately, we may employ the empirical formula $\gamma = \bar{\gamma} - aT$, where the values of $\bar{\gamma}$ and a for a given gas may be computed. In this case we may write the following formula:

$$u_0 = u_{initial} + \int_{T}^{T_{initial}} \sqrt{\frac{c_p c_v}{RT}}\,dT \qquad (2a)$$

Since $c_v = R/(\bar{\gamma}-1-aT)$ and $c_p = R(\bar{\gamma}-aT)/(\bar{\gamma}-1-aT)$, integration gives:

$$u_0 = \frac{D}{\gamma_{initial}+1} + 2\sqrt{\frac{R}{a}}\left[\arcsin\sqrt{\frac{\gamma-\gamma_{initial}}{\gamma_{initial}}} - \arcsin\sqrt{\frac{\bar{\gamma}-\gamma}{\gamma}} + \right.$$
$$\left. + \frac{1}{\sqrt{\bar{\gamma}-1}}\left(\mathrm{Arth}\sqrt{\frac{(\bar{\gamma}-1)\,\gamma_{initial}}{\bar{\gamma}-\gamma_{initial}}} - \mathrm{Arth}\sqrt{\frac{(\bar{\gamma}-1)\,\gamma}{\bar{\gamma}-\gamma}}\right)\right] \qquad (4b)$$

The difference in the rates, as determined by formulae (4) and (4b), is not very large, although it still amounts to as much as 8 per cent of the actual value of the rate.

The velocity of the shock-wave and the temperature at its front were determined by the formulae:

$$D_{shock} = \frac{\gamma_{atm}+1}{2}\,u_x \qquad (7)$$

$$T_{shock} = \frac{\gamma_{atm}-1}{2}\,u_x^2 \qquad (7a)$$

The velocity of flow into the atmosphere has to be computed by the method of successive approximations or graphically, proceeding as before from the condition that the pressure at the front of the shock-wave should be equal to the residual pressure of the detonation products. The results of our computations are summarized in the following table.

Gaseous mixture	D (in m/sec.)	T_{initial} (in °K)	v_{initial} (in m/sec.)	p_{initial} (in kg/cm²)	u_0 (in m/sec.)	u_x (in m/sec.)	p_{shock} (in kg/cm²)	D_{shock} (in m/sec.)	T_{shock} (in °K)
$2H_2+O_2$	2820	4600	1330	19	21500	1350	19	1600	1000
$2H_2+O_2+N_2$	2400	3900	1100	19	15000	1200	18	1400	900
$2H_2+O_2+3N_2$	2060	2700	930	18	11600	1000	16	1200	850
$2H_2+O_2+5N_2$	1820	2700	840	17	7900	900	14	1100	770

It is evident that the limiting velocities that arise in an unsteady flow may be quite large, well above those of a stationary process. For the case of flow into the atmosphere, the velocities have no time to increase owing to the low density of the detonating gas, and are damped by the shock-wave that forms.

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PHYSICAL CHEMISTRY

ON THE EXPERIMENTAL DETERMINATION OF THE PROBABILITY OF THE PERISHING OF ACTIVE CENTRES

By A. B. NALBANDJAN

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Mention has frequently been made in the literature of the fact that the introduction of pieces of metal wires into quartz or glass vessels markedly raises the lower ignition limit of combustible mixtures. At the same time the latter has been observed to shift hundreds of degrees into the region of high temperatures. This phenomenon was qualitatively explained by the high value of the probability of the perishing of an active centre ε on the metal surfaces. Quantitatively, however, it was still unclear in what way the introduction of such an insignificant surface, amounting to 1 or 2 per cent of the total surface of the reaction vessel, was able to change so markedly the lower limit.

In a paper devoted to the rupture of chains on the surface of solids, N. N. Semenov [1] develops in detail the theory of the lower limit of ignition, taking into account the probability of the perishing of active centres on the walls. In this paper, as one of the applications of the theory, the interesting question of the influence of metal wires introduced into a glass reaction vessel along its axis is analysed quantitatively for the first time. In his work Semenov assuming, on the one hand, that $\varepsilon = 10^{-5}$ or less on treated glass surfaces and, on the other, that $\varepsilon = 10^{-2}$ or greater on metallic surfaces, arrived at the following expression for the square of the product of the pressure at the lower limit and the diameter of the glass vessel when wires of diverse diameter are present:

$$(pd)_k^2 = \frac{4B}{\lg (d/\Delta)} \qquad (1)$$

where d is the diameter of the glass reaction vessel, Δ is the diameter of the wire, and B is a constant for each given temperature.

In the course of our work we obtained a number of data offering the possibility of checking expression (1). As will be shown below, these data corroborate Semenov's conclusions and thus make it possible to determine quantitatively by simple experiments the probability of the rupture of a chain ε on various surfaces. Our experiments were carried out in an annealed pyrex vessel 55 mm in diameter and 90 mm long. Through a ground glass joint sealed to the end of the vessel stainless steel wires 80 mm long and 0.2, 0.1 and 0.045 cm in diameter were introduced in succession. The lower limit of self-ignition by the mixture $2H_2 + O_2$ was obtained as usual by the rapid introduction of the reacting mixture from a gas measuring vessel.

Before the experiments were started, the reaction vessel was subjected to training for a period of several days by ignition of the hydrogen-oxygen mixture.

Training of the empty vessel was considered completed when an extremely low and well reproducible lower limit was obtained. After this the wires were introduced into the reaction vessel along its axis, and measurements of the lower limit of ignition in the presence of the latter were carried out.

To avoid the influence of the upper limit of ignition, the measurements were carried out at high temperatures, far from the cape of self-ignition of the hydrogen-oxygen mixture. Values of the limiting pressures measured at $T = 464°$ C are given in the table. The same measurements with analogous results were made at $T = 449°$C.

Lower Limit of Ignition of $2H_2+O_2$ at $T=464°$ C

Diameter of wire (Δ in cm)	Lower limit of ignition p_1 (in mm of Hg)
0	0.95
0.2	4.20
0.1	3.80
0.045	3.10

Knowing the value of p_1, d and Δ and applying formula (1), it is easy to compute the value of B from the data given in the table. Thus, for the wires

$$\Delta = 0.2 \text{ cm}, \quad B = 191$$
$$\Delta = 0.1 \text{ cm}, \quad B = 189$$
$$\Delta = 0.045 \text{ cm}, \quad B = 152$$

As we see from the experiments with the wires, $\Delta = 0.2$ and 0.1 cm, B is indeed practically constant at the given temperature. A certain deviation is observed for the wire 0.045 cm in diameter *. The mean value for B from three experiments with three different wires is equal to $B_{mean} = 180$.

On the other hand, the absolute value of B may be computed from the curve for the lower limit of ignition that we obtained in an iron vessel ([2]). As Semenov has shown ([1]) in the case where ε is sufficiently large, the product of the critical pressure at the lower limit and the diameter of the reaction vessel should satisfy the condition

$$(pd)_k = 4.8\sqrt{B} \qquad (2)$$

Knowing that the diameter of the iron vessel is equal to 2.5 cm and that $p_1 = 16.3$, 14.4 and 12 mm of Hg at 590, 612 and 650° C, respectively, we can by extrapolation compute the value of B at 464° C. At $T = 464°$ C, $B = 360$.

As we see, the mean value of B that we computed by means of formula (1) from the data for the lower limit of self-ignition in a pyrex vessel in the presence of wires of diverse diameter, is half as large as the value of B computed by formula (2) from the data for the lower limit in an iron vessel. However, both values are of the same order of magnitude. It is highly probable that the quantitative deviation is connected with the inaccuracy in extrapolation and with the inaccuracy in determininig the lower limit in the iron vessel, owing to the presence of a certain heterogeneous reaction which leads to the formation of noticeable quantities of water vapour.

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* As can be easily seen from formula (1), to obtain $B = 190$ for the case of the wire $\Delta = 0.045$ cm, the pressure at the lower limit must be equal to 3.35 mm instead of the experimentally observed value of 3.1 mm. Such a deviation (7 per cent) lies within the limits of experimental error.

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GEOLOGY

ON THE GEOLOGICAL STRUCTURE AND REGULARITIES IN THE DISTRIBUTION OF MINERALIZATION IN THE DJAMANKTAI ORE FIELD IN CENTRAL KARA-TAU

By V. V. GALITZKY

(Communicated by V. A. Obruchev, Member of the Academy, 6. VII. 1944)

The Djamanktai ore field, located east of the well-known Turlan lead-zinc deposit, is one of the most promising ore fields of Central Kara-tau. It is here that has been found the greatest number of ore centres, and in spite of the complexity of the geological structure the nappe structure which is so characteristic of the Kara-tau is well pronounced here. The relation between the geological structure and ore mineralization is more readily traceable in this field than in any other part of Central Kara-tau.

The stratigraphic section of the region is composed of three different lithological complexes: competent arkose sandstones of the Devonian, incompetent clay-carbonate Famennian deposits, and competent carbonate Lower Carboniferous deposits, a general characteristic of which is given in the previous papers by the present author ([2],[3]).

In the process of deposition the Kara-Bulak outcrop of Devonian sandstones of considerable size presented a concave lens of an alluvial fan (delta) with a monoclinal primary dip of the beds. The total thickness of the separate beds of the series, 3700 m, seems to exceed considerably the vertical thickness of the lens. An unconformity between the arkoses and the overlying argillites has been established.

The Famennian deposits of the ore field represented by the Turlan facies are distinguished from the other areas of Central Kara-tau by a greater thickness (to 1300 m), mainly at the expense of a sharp increase in the thickness of the clay horizons of the section, this thickness being twice that of the Turlan section.

The stratigraphic sections of Lower Carboniferous lying on Famennian deposits in a tectonic nappe differ considerably in adjacent blocks of the nappe. The facies of the section of Lower Tournaisian deposits in individual blocks have been found to be rather uniform, the thickness of the horizons being highly variable. In the horizon of black dolomites Ctb an abundant but monotonous fauna has been collected represented by *Spirifer* (*Cyrtospirifer*) *kurban* Nal., common for the Etroegunt deposits of Kazakhstan. In the upper portion of the Lower Tournaisian the presence of thick lenticular bodies of sedimentary breccias resembling very much the Dinantian breccias of Belgium has been recorded([5]). Between the Lower and the Middle Tournaisian a hidden unconformity has been discovered.

The section of the Middle Tournaisian is represented by diverse facies of rapidly alternating thickness and strike. The facies of black flag finely crystalline limestones, containing a large number of thin black flint lenses, is characterized

by a brachiopod fauna*: *Leptaena analogua* Phill., *Schizophoria* cf. *chouteanensis* Well., *Rhipidomella* cf. *michelini* L'Ev., *Prod.* (*Plicatifera*) sp. ind., *Brachythyris suborbicularis* Hall., *Tylothyris laminosus* M. Coy, *Syringothyris* ex gr. *cuspidata* Mart., *Syringothyris* cf. *carteri* Winch., *Athyris* sp. ind., *Reticularia* cf. *pseudolineata* Hall., *Camarotoechia pleurodon* Phill., *Spirifer* sp. (nov. ?). In addition, *Platyceras* sp., *Orthotetes* sp., *Syringopora uralica* Stuck. are present.

Somewhat lower stratigraphically the facies of light-grey reef limestones, with a rare bedded jointing, consisting largely of remains of calcareous algae, contains the following fauna: *Leptaena analogua* Phill., *Prod.* (*Plicatifera*) aff. *aktaicus* Nal., *Prod.* (*Dictyoclostus*) *djamanctaicus* sp. nov., *Prod.* (*Pustula*) cf. *subpustulosa* Thom., *Lamellispirifer roemerianus* Kon., *Spirifer centronatus* Winch., *Brachythyris* sp., *Reticularia cooperensis* Swall., *Reticularia* cf. *lineata* Mart., *Martinia* sp., *Camarotoechia* sp.; then *Orthotetes* sp. and pelecypods *Leiopteria*, *Aviculopecten* and *Crinoidea*.

In the same reef limestones, but stratigraphically on the same level as the fauna of black flag limestones, the following brachiopods have been collected: *Leptaena analogua* Phill., *Schizophoria* cf. *chouteanensis* Well., *Prod.* (*Dictyoclostus*) *djamanctaicus* sp. n., *Prod.* (*Plicatifera*) sp. ind., *Productella* (?) *semicircularis* sp. nov., *Prod.* (*Pustula*) sp., *Spirifer* cf. *striatiformis* Meek, *Brachythyris* sp. (cf. *ovalis* Phill.), *Brachythyris* cf. *suborbicularis* Hall., *Spirifer* cf. *centronatus* Winch., *Plicatifera* cf. *aktaicus* Nal., *Linoproductus* aff. *sampsoni* Well., *Camarotoechia* cf. *cidertensis* Nal., *Orbiculoides* sp., pelecypods *Lamondia*, *Leiopteria*, *Parallelodon*, trilobites, stems of *Crinoidea*, *Orthoceras* cf. *neglectum* Kon.

The facies of carbonate-clay shales is characterized by a fauna of goniatites from the group *Protocanites lyoni* Meek and Worthen.

Near the Taldy-Bulak deposit a section of Lower Carboniferous deposits has been found in a tilted up attitude. This is the first finding in the Kara-tau; the age relations have been corroborated by palaeontological evidence. In the core of a false anticline under Lower Tournaisian deposits, lie Middle Tournaisian deposits with goniatites ex gr. *Protoconites lyoni* Meek and Worthen.

In the Djamanktai ore field, as well as within the remaining area of Central Kara-tau, the three complexes of rocks, differing as to their lithology, are separated by two regional zones of strip overthrusts formed during the Variscian phase of folding: the lower, between Devonian argillites and Famennian deposits, and the upper, between Famennian and Lower Carboniferous deposits. The shape and size of the brachyanticlinal exposure of arkose sandstones, and those of the Famennian deposits are quite incompatible. No evidence is to the appearance of sandstones in the cores of anticlinal folds of the Famennian deposits—the cores are filled with a brecciated material of argillites. The disturbances present in the Famennian deposits do not extend to the sandstones, but all of them vanish in the argillite breccias. The relations between the sandstones are similar to those between the Mesozoic and the crystalline basement in the Jurassic mountains; they go well to illustrate the correctness of the theoretical conceptions by Bubnoff(1) concerning the relation between isoclinal folds and the underlying basement.

The complication of folding observed from the lower horizons of Famennian deposits towards the upper ones, to the boundary with the Lower Carboniferous, is a consequence of a displacement of the nappe of incompetent Lower Carboniferous deposits along incompetent Famennian deposits, and the folds of the latter are principally drag folds.

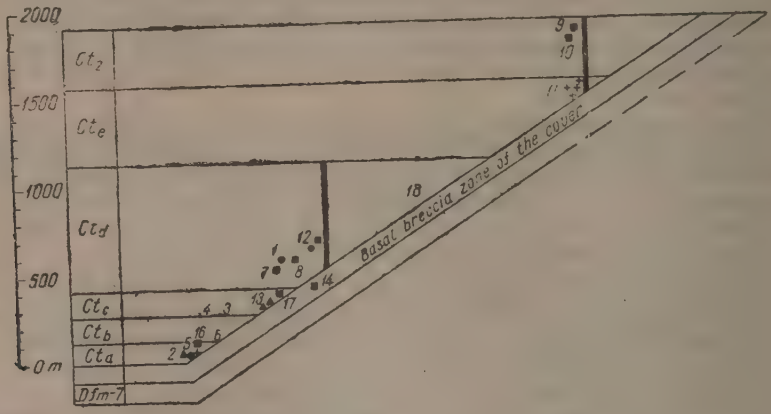
Within the Djamanktai ore field the nappe of Lower Carboniferous deposits is broken into separate tectonic blocks, different horizons of the latter lying on the upper horizons of Famennian deposits. Occasionally, there occurs shearing of the lower horizons over rather extensive areas (300—400 m) from the base of the Carboniferous column. In a number of nappe inliers and in the zone of strip overthrust, Devonian argillites are exposed. The disturbances separating the blocks of

* The brachiopods have been determined by I. L. Timofeeva; the goniatites, by L.S. Librovich; the corals, by I. I. Gorsky.

the nappe of Carboniferous deposits do not extend below the zone of the strip overthrust in the Famennian. The disturbances in the Famennian deposits do not continue through the same zone into the upper nappe. Among the blocks there is a tilted recumbent fold, the Taldy-Bulak false anticline.

Each block taken separately is exotic. A comparison with the adjacent blocks makes this exotic nature still more conspicuous. The present arrangement of the blocks is not likely to have been attained without considerable horizontal shifts.

In contradistinction to the Turlan field previously studied by the author ([2–4]), characteristic of the Djamanktai ore field is the fact that on one and the same



Distribution of ore centres in the cumulative stratigraphical section, and with respect to basal breccia zone of the nappe and steep disturbances separating the blocks. Scale of horizontal and vertical distances between separate ore centres and disturbances and vertical scale for the whole graph 1 : 40000. Distance between centres without scale. Symbols: black circle, intercalations of galenite and residual galenite in brown ironstones: black triangle, brown ironstones: cross, fluorite intercalations: square, lead-zinc deposits; 7—old grooves: *8*, Djamanktai I; *9*. Djamanktai II; *10*. Djamanktai III; *12*, «20th of October»; *14*. Kuruktai; *16*, Taldy-Bulak; *17*, Karabadam

horizon of Famennian deposits (Dfm 7) the blocks of the Carboniferous nappe rest with their different horizons, the basement of the allochthonous deposits having been cut of. In the Turlan field the allochthon shears away the autochthonous structures. The blocks of the Djamanktai field are of small size; they are fragments of some larger structures.

In comparing the peculiarities in the structure of the Djamanktai field with the structure of the surrounding areas of Central Kara-tau and considering the regional structure as a whole, the author has come to the conclusion that the movement of the nappes along strip overthrusts proceeded from north-west to south-east. The blocks of the Carboniferous deposits of the Djamanktai field have been shifted through the Kara-Bulak exposure of arkose sandstones of the Devonian. As a result, the basement of some of the blocks of the nappe have been worn off. The Famennian deposits appear to have been stripped from the sandstones and partly entrained by the lower surface of some of the blocks, which fact accounts for the presence of blocks of Famennian limestones and argillites in the windows of the nappe.

The lead-zinc deposits of the field belong to telethermal cryptobatholitic ores. Of the eighteen ore centres known one is located in the upper horizon of Famennian deposits; and the overwhelming majority, in different horizons of Lower Carboniferous deposits, which is indicative of the small importance of the selective properties of any individual horizon of the Carboniferous. The lithological composition and thermodynamic conditions in the Carboniferous deposits are more fa-

vourable for the formation of ore occurrences than in the underlying Famennian. The static load upon the incompetent Famennian clay deposits was considerably greater than that on the Carboniferous: hence the ways open to the circulation of hydrothermae in the former were developed to a greater extent than in the latter.

The ore occurrences are located in the limbs of the synclinal folds of the blocks of the nappe, below the points of bend of synclinal to anticlinal limbs, in the hanging limb of the zone of the strip overthrust, and within a comparatively small distance from the latter. The geological control of the deposits is determined by the distinctly pronounced tectonic factor.

The statement of Newhouse ([7]) as to the ore deposits being confined to anticlinal folds, supported by Labazin ([6]) for the Kara-tau, are not correct; the sole fact is beyond any doubt, that the hydrothermal solutions move from zones of greater pressure to those of lower pressure.

As a matter of fact, the majority of the deposits of the Djamanktai field and of the whole Kara-tau area are confined to limbs of synclines, the movement of the hydrothermae having been directed from the hinges of synclines towards their limbs ([4]).

The anticlinal bends, equidimensional with the synclines comprising the deposits in the Kara-tau region, have been disrupted and removed by denudation, while the Variscian structure was still in process of shaping, prior to the formation of the deposits; they do not exist, and for this single reason cannot be promising in the search for deposits.

Fortunately, the present level of erosion coincided with the upper portion of the thermodynamical zone, in which the deposits were formed. That is why most of the Kara-tau deposits are characterized by small dimensions of outcrops, and the possibility of finding new semi-blind and blind ore bodies in the synclinal zones of the Middle Palaeozoic in the Kara-tau area is very high.

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GEOLOGY

ON THE PRESENCE OF GYPSUM IN THE LOWER CRETACEOUS DEPOSITS OF THE VOLGA REGION

By M. V. CHURINOV

(Communicated by F. P. Savarensky, Member of the Academy, 4. VII. 1944)

The presence of gypsum in rocks may be of interest when solving a number of practical and purely theoretical problems. The occurrence of syngenetic gypsum illustrates the peculiar palaeogeographical conditions of deposition of the sediments.

The occurrence of gypsum in the Lower Cretaceous deposits has been reported by many geologists working in the Volga region, the lithological characteristics of the rocks being invariably accompanied by a record of the presence of pyrites and gypsum. Late Prof. E. V. Milanovsky [1], one of the most eminent authorities in the geology of the Volga region, says: «In the Middle Volga region the Valanginian is overlain by thick beds of black and dark-grey clays with pyrite, gypsum crystals and large ellipsoid ferro-calcareous concretions of a peculiar structure».

Further he says: «It is in the Hauterivian that a quiet, rather deep sea was formed, which in the northern part of the area described (the Gorky, Kuibyshev and northern part of the Saratov region) deposited thick beds of clays rich in gypsum and pyrites».

In 1939 the present author, with the object of studying landslides, participated in the engineering-geological work carried on by the administration of the Kuibyshev water-power plant in the region of the town of Ulyanovsk; he succeded in collecting interesting material characterizing the structure, composition and properties of the Lower Cretaceous deposits of this region. An examination of numerous cores, obtained in drilling holes by means of devices for sampling undisturbed soil, as well as a direct inspection of Lower Cretaceous rocks *in situ* in deep test-pits exposing the whole thickness of Lower Cretaceous deposits from the Albian to the Hauterivian, throw new light upon the occurrence of gypsum in the Lower Cretaceous of the Volga region.

Both on the dividing plateau and on the slopes down to the Volga and Sviaga, the country rocks have been to a varying degree altered by weathering processes down to a certain depth from the day surface; and the weathering zone within which the degree of alteration of the rocks gradually diminishes downwards is everywhere distinctly pronounced. The thickness of the weathering zone on the plateau attains 10—15 m. In the upper parts of the slope, having a maximum gradient, the thickness of the zone of weathering does not exceed 3—4 m owing to a constant annual displacement of the superficial, most strongly weathered portion.

Apart stands the area of landslide accumulations in the middle and lower parts of the slope, where for some time the Cretaceous rocks sliding downward are being accumulated; these have been weakened by weathering and as a result

of movement show a complete distortion of structure and peculiar textural features, strikingly distinguishing them from the eluvium of the country rocks on the dividing plateau.

The thickness of the landslide accumulations on the Volga slope attains 25 m.

Attaining the zone of weathering, the country rocks begin to suffer some changes of their chemical and mineralogical composition, becoming enriched in new minerals, as a result of which their physical and mechanical properties begin to change, too.

Glauconite and especially marcasite, under the influence of oxidation, pass into iron hydroxide, giving to the weathered rocks a characteristic yellow-brown colour. As a consequence of the decomposition of marcasite, in the zone of weathering there appears gypsum in its different modifications: coarsely crystalline, finely crystalline and microcrystalline. From the author's observations in the test-pits, and from personal information of K. A. Kabanov, a local authority in the history of the region, who has for many years been engaged in collecting crystals of gypsum on the bank of the Volga, coarsely crystalline gypsum occurs in black, slightly altered Lower Cretaceous clays only, *i. e.* in the lowermost portion of the weathering zone. The area of development of coarsely crystalline gypsum is characterized by a small thickness, not exceeding 0.3—0.5 m, and an insignificant depth of occurrence of an order of 1—5 m. In the higher horizons of the weathering zone only finely crystalline and microcrystalline gypsum is to be found.

A microscopic examination of the Lower Cretaceous clays in thin-sections, comprising both the weathering zone and the country rocks, known to have suffered no weathering whatever, had fully confirmed the above-mentioned regularity in the vertical distribution of gypsum and its morphological peculiarities. Thin-sections of clays lying below the weathering zone furnish indications as to the presence of fresh marcasite, while in the zone of weathering the clays always contain microcrystalline gypsum formed in fissures, and, as a rule, marcasite is absent from the upper portions of the zone of weathering.

Analyses of water extracts from Lower Cretaceous rocks *in situ* have shown the ratio $Na/(Ca+Mg)$ to be, as a rule, above unity, the absolute content of Ca and Mg being comparatively low. Analyses of extracts from the weathering zone have revealed that the Ca and Mg content sharply increases, and the ratio $Na/(Ca+Mg)$ becomes below unity.

Moreover, in the latter case the sulphate content sharply increases as compared to that of other salts and to the absolute content of sulphates in the rocks unaffected by weathering.

Large gypsum grains originate from the places of marcasite accumulation, when Lower Cretaceous clays come to the weathering zone. Here, under the influence of the oxygen and carbon dioxide of surface waters penetrating along fissures, oxidation of marcasite takes place, accompanied with the formation of ferric iron and sulphuric acid. The formation of gypsum is promoted mainly by the oxidation and dissolution of calcareo-siderite concretions which supply most of the calcium, along with the calcium contained in the water percolating from the surface.

This is supported by the presence of semi-decomposed calcareous siderite, usually observed near the places of occurrence of gypsum (information obtained from K. A. Kabanov).

The appearance and growth of large gypsum crystals take place in black, as yet slightly altered clays, where the processes of weathering are little active and where, owing to an insignificant number of fractures, the circulation of waters is very slight.

Under the influence of denudation of the slope, the depth of occurrence of coarsely crystalline gypsum diminishes, and at the same time the activity of the processes of weathering becomes increasingly greater. The gypsum crystals begin to decompose and dissolve enriching the rocks of the weathering zone with sulphate salts.

Near the day surface, where the processes of weathering are the most vigorous, the surface waters circulating along fractures are enriched with calcium sulphates and become saturated with these to the utmost; in the narrower fractures or closed voids crystallization of gypsum takes place in this case in its micro- and finely crystalline modifications, as a result of their comparatively short growth from an oversaturated solution.

All the above data on the presence of gypsum in the Lower Cretaceous deposits are characteristic not only of the town of Ulyanovsk. An examination of the geological materials in Senghiley, Volsk and Kashpir confirms the regularity observed in the occurrence of gypsum only within the weathering zone of Lower Cretaceous rocks.

The mineralogical composition of Lower Cretaceous clays containing marcasite and calcareous siderites is evidence of a reducing medium in which the given deposits were formed, which excludes the possibility of syngenetic gypsum having been formed.

As well known, gypsum is a highly hydrated mineral, capable of being formed only in the presence of free oxygen and under normal (low) temperature conditions.

Thus, it should be pointed out that gypsum is not inherent in the Lower Cretaceous rocks of the Lower Volga region *in situ*, and that it is formed and occurs only in the zone of weathering of these rocks as a result of oxidation and decomposition of the pyrites developed in clays. In this connexion the question as to the presence of gypsum in the Upper Jurassic deposits of the Volga region, which very closely resemble the Lower Cretaceous ones in their lithological composition and external habit, should be subjected to critical revision.

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ECOLOGY

PRINCIPAL PROPERTIES OF INJURIOUS INSECTS SURVIVING TO FIELD CROP-ROTATION

By M. S. GHILAROV

(Communicated by I. I. Schmalhausen, Member of the Academy, 23. IV. 1944)

Within the range of its distribution every species occupies definite habitats which fulfil specific conditions. The environmental conditions in a given habitat must be in accordance with the «ecological standard» of the species.

The range of fluctuation of environmental factors in a station inhabited by a given species, which permit of its permanent existence there, is limited by the degree of «ecological plasticity» of the species.

The adaptation of the population of a given species to the fluctuations of environmental conditions in definite habitats can take place either as a result of adaptive reactions (phenotypical, modificational changes, behaviour, *etc.*) of the organisms in the course of each ontogenesis (within the range of the «norms of reaction» of a species), or, in the case of gradual changes, by way of the selection of mutations better adapted to the existence under altered conditions.

However, the variations in the environmental conditions in the habitat occupied by a population, especially at the periphery of the species' area of distribution, may exceed the limits permitted by the ecological plasticity (the range of ecological standard) of the population's components. In this case, if the population there is unable to change the habitat, it must perish and its «niche» in the biocoenosis inhabiting a given biotop must be occupied by a population of an other species.

Dispersal is therefore along with propagation the principal function of the specific life of the organism.

The lower the ecological plasticity of the organism and the narrower its ecological standard, the more important for its survival under varying conditions is the function of dispersal. Data secured from experience of agricultural entomology go well to substantiate the above-said.

One of typical features of modern farming is the crop-rotation, owing to which both the specific composition and the character of vegetation changes, as a rule, every year over more or less extensive areas (crop-rotation fields) determined by the kind of the crop raised in a given year.

The complexes of phytophagous insects in the biocoenoses of arable lands are, therefore, forming at the expense of such species, as are able not only to feed on cultivated plants, but also to endure the brusque changes in the character and composition of vegetation in a definite habitat of a population.

Among species of phytophagous insects which are able to feed on cultivated crops and propagate in masses under field conditions, are to be found both species of wide ecological plasticity as to the choice of food-plants (poly-

phagous pests, eurytrophic plantivorous insects) and species of a very narrow ecological standard with respect to food-plants («specialized» pests, monophagous and oligophagous stenotrophic plantivorous insects).

An analysis of the biological properties of the representatives of both groups of insects mentioned above, which are of considerable economic importance as pests of cultivated plants, has shown that the degree of the stenophagy is connected with the ability of the species to spread and with the duration of its life-cycle.

To the majority of insects is peculiar the separation in the ontogenesis of the nutrition function (accumulation of plastic and energetic matter) from that of dispersal.

The former is characteristic largely of the larval stage, the latter (in the case of the active dispersal), of the adults *.

Most insects (and in particular injurious ones) are characterized by a low ability to migration in the larval stage, especially as compared to that of imagines. In the cases where the duration of the larval stage exceeds one year, or where the larval development embraces two periods of vegetation (in temperate climate), only such forms are able to survive under conditions of crop-rotations, as are able to endure the brusque changes of food-plants, determined by the changes of crops, *i. e.* polyphagous species of high ecological plasticity with respect to the choice of food.

Such are the wireworms (the larvae of *Elateridae* beetles), for example, *Agriotes* spp. or *Selatosomus* spp., widely distributed in arable lands in the central regions of Russia, whose life-cycle is about 5 years long. The wireworms are known to attack all field crops cultivated in Russia.

Well-expressed polyphagy is peculiar to the injurious *Scarabaeidae* larvae, whose life-cycle lasts 2 years (*Anisoplia* spp.), 3 years (*Amphimallon solstitialis* L.), 4 years (*Pentodon idiota* Hbst.), or even 4—5 years (*Melolontha*).

Highly polyphagous are the *Alleculidae* larvae (*Omophlus proteus* Kitsch., *Podonta daghestanica* Reitt.) which develop during two years; they are widely distributed in the fields of the southern part of the forest-steppe zone and in the steppe zone of the Ukrainian SSR.

Polyphagous are also the larvae of *Tenebrionidae* beetles of the genus *Blaps*, which develop within about 16 months, and are known as dangerous pests of many field crop varieties in the steppe zone.

The larvae of the crane-flies (*Tipulidae*) which cause heavy losses to the most various components of field crop-rotations in the districts of high humidity (the western provinces of this country), emerge from the eggs in August-September, feed on underground parts of various plants till late in autumn, hibernate and resume feeding in the next spring, pupating in July-August (*Tipula paludosa* Mg.). Thus, though the development of these common and widely distributed pests lasts less than one year, it extends to two periods of plant vegetation.

The so-called «gray weevil» or «minor sugar-beet weevil» (*Tanymecus pallidus* F.) which has a two-year life-cycle, is known to attack a variety of cultivated dicotyledonous plants.

A great number of analogous examples can be cited.

All injurious insects widely distributed in arable lands, which remain in the larval stage during several vegetation periods (over one year), are represented by species remarkable for their polyphagy.

This phenomenon can be explained in the following way: the oligophagous larvae which are unable to migrate to great distances in the case of sudden changes of vegetation cover produced by crop-rotation, are exterminated in large num-

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* Only in relatively rare cases of regressive development the function of dispersal is carried out by juvenile stages (for instance, by the larvae in *Coccidae*). In such cases the passive transport of individuals is of prevalent importance in the dispersal.

bers, whereas such larvae, as are able to endure the changes of food-plants, *viz.* polyphagous ones, survive in crop-rotation.

Highly polyphagous are also the pests of field crops, which, though having a life-cycle shorter than one year, are not able to migrate over large distances, say to an adjoining field during any development stage.

Such is, for example, the «white root-aphid» (*Trifidaphis phaseoli* Pass.), the well-known pest of the majority of cultivated and weedy herbaceous dicotyledonous plants. No winged individuals of this insect are discovered under field conditions of the forest-steppe regions of the Ukraine (5).

The species of injurious insects of a narrow ecological standard as to the choice of food-plants, which are lacking a well expressed ability to disperse, or in which the periodicity of the dispersing stage (the imaginal one in all the field-crop pests) has a frequency lower than that of the changes of the vegetable cover involved in crop-rotation, are destined to extinction. At any rate they cannot find favorable conditions for large-scale propagation and cannot become pests of any economic importance.

Even if highly adapted to the cultivated plant they attack, such species can but rarely be found under conditions of regular crop-rotation and cannot be important pests. Such are, for example, the «ground longicorns» of the genus *Dorcadion* (*D. carinatum* Pall., and allied spp.) in the Ukraine, whose larvae feed on the roots of *Gramineae* and develop in the soil during two years, whereas the adult beetles are wingless.

To the same group of pests belong the «sod webworms» (*Crambus*), «southern stem cutworm» (*Oria musculosa* Hb.), which feed on small-grains; these caterpillars develop during two vegetation periods. These pests can be of some importance and played some part in the case of backward farming methods under conditions of permanent cultivation of cereals, or of contamination of the crops with graminean weeds (1).

To the same group of pests belong the *Carabidae* larvae of the genus *Zabrus*, affecting winter crops in the Ukraine. The *Zabrus* beetles accumulate, feed and deposite their eggs in fields under grain-crops. The larvae which hatch at the time of the harvesting of cereal crops survive only at plots with vegetating cereals (sprouts of shed kernels or of winter crops) upon which they feed. Such winter-wheat as is not sown after some other grain crops, does not suffer noticeably from these pests (2).

Thus, oligophagous injurious insects of low mobility, whose life-cycle continues for several periods of vegetation under the conditions of crop-rotation, are of no importance, though they may cause considerable damage in the case of backward forms of farming (permanent cultivation of the same crop, *etc.*).

All the monophagous and oligophagous insects which are highly important pests of field crops, and occur in great numbers on arable lands, are represented by forms whose larvae complete their growth within a time not exceeding one period of vegetation, and whose imaginal stage is able to disperse (usually by flight) over more or less considerable distances.

Such are the most injurious specific pests of cereals: the Hessian fly (*Mayetiola destructor* Say.), the frit-fly (*Oscinosoma frit* L.), other cereal flies (*Chloropidae*), wheat stem sowflies (*Cephus*), *etc.* To these belong also the oligophagous insects attacking the sugar-beet: the sugar-beet weevil (*Bothynoderes punctiventris* Germ.), the sugar-beet «tortoise beetle» (*Cassida nebulosa* L.), the sugar-beet flea-beetle (*Chaetocnema concinna* Marsch), and similar pests specific to the sugar-beet.

The sam erefers to the specialized pests of leguminous plants, for example, the pea-weevil (*Bruchus pisi* L.) on peas, *Sitona* spp. on different varieties of annual and perennial *Leguminosae*, *Apion* spp. on red clover, *Phytonomus variabilis* Hbst. on lucerne, *etc.*

Thus, owing to the sudden changes of vegetable cover yearly produced by the agricultural activity of man on arable lands, stenotrophous plantivorous insects are selected, possessing a short life-cycle and a high ability to dispersal.

Forms with a low ability to migration (and with perennial life-cycle) are represented exclusively by polyphagi*.

Under the conditions of crop rotation, including the fields kept clean, among the forms with reduced mobility and perennial life-cycle, such are selected as are able to survive in the absence of living plants during a considerable period (3) and to feed on decaying vegetable remains.

Thus, the ability of a species to dispersal is a peculiarity which may compensate for its insufficient plasticity under the conditions of varying environment.

The above analysis shows that under varying conditions of environment (even in the case of a single varying factor) the forms surviving in a given station are selected not only for a single feature, but for a whole complex of features. The latter must not necessarily be in a direct relation to the varying factor of the environment (degree of euryphagy and variation of specific composition of vegetation cover), but may also be indirectly related to this factor (ability to dispersal).

The practical importance of this analysis consists in the following: it permits to establish the rôle of crop-rotation in the control of an insect pest on the basis of determination of fundamental biological features which can readily be recorded. The crop-rotation is of major importance in the control of stenotrophic species of low mobility and of perennial life-cycle(2,4). The increasing ability to dispersal, or a higher degree of polyphagy interfere with the effect of crop-rotation; the effect is practically nil in the case of actively moving polyphagous insects with a short life-cycle.

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* Of course, among the polyphagous insects affecting the field crops there is a great number of species whose life-cycle lasts one year or less. To these belong the majority of injurious pests, such as *Acrididae*, webworm (*Loxostege sticticalis* L.), the cutworm (*Euxoa segetum* Schiff.) and others.

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PLANT PHYSIOLOGY

CRITICAL PERIODS IN THE UPTAKE OF POTASSIUM BY SUGAR-BEET

By T. T. DEMIDENKO

(Communicated by A. A. Richter, Member of the Academy, 12. VII. 1944)

In order that a plant may grow and develop normally it is necessary that all the factors of growth be available in their most advantageous combination. A total or temporary elimination of any such factor will produce a negative effect upon the general development of the plant. Let us dwell here at some length upon the mineral nutrition of plants, *viz.* upon the critical periods of the Ca uptake by sugar-beet.

A number of students have arrived at the conclusion that potassium is contained in plants in the ionic form and that it is deposited in those tissues where intense assimilation is taking place. Such vegetable tissues as contain more potassium are higher in water, as well; the effect of potassium is therefore to produce an intense swelling of the colloids and to increase the water-fixing capacity of the latter, which facilitates the translocation of sugars from the leaves to the roots.

It has been demonstrated by number of authors ([1-5]) that owing to its high mobility and radioactive properties potassium serves as an activizer of oxidizing reactions of biochemical processes.

Experiments were undertaken by the present author with the aim to determine the critical periods in potassium nutrition of sugar-beet. This question arose in the course of industrial testing of nutrient substances supplied to this crop as staple fertilizers or as extra-diet.

Among the nutrient elements absorbed by sugar-beet potassium is the one consumed by the plant in the largest quantity. In many a region where this crop is raised potassium is a most desirable fertilizer. In order to determine the dates at which the supply of potassium gives the best effect, experiments were carried out on water-cultures grown in vessels of a 10 l capacity.

Within the period of vegetation the solutions were renewed five times, every fifteenth day. Beloussov's nutrient solution was made use of. In order that potassium be excluded from the solution, phosphoric acid was introduced in the form of calcium biphosphate. When $CaHPO_4$ was supplied, the reaction of the nutrient solution became an alkaline one, and therefore, in order to maintain the reaction within a range of 5.5—6.0 pH, 0.1 *N* sulphuric acid has to be added.

The results of the experiment are given in the table.

Sugar-beet is storing potassium intensely while young. The plant goes through a critical period between the appearance of 4 pairs of leaves till the appearance of 7 pairs. To its removal from potassium diet at any period of growth the sugar-beet answers by a decrease in the sugar content both in absolute and relative figures.

Yield and Composition of Sugar-Beet as Affected by Potassium Starvation

No. of runs	Experiment scheme					Weight in g		Root	Weight of total plant, g	Sugar, %	Harmful nitrogen, %	Colloids, %	Dry weight of plant, root and leaf, g	Absorbed in g		
	planting	4 pairs of leaves	7 pairs of leaves	10 pairs of leaves	13 pairs of leaves	root	leaf							N	K_2O	P_2O_5
1	H	H	H	H	H	575	540	68	1183	16.9	0.203	2.436	169.6	3.192	5.513	2.013
2	H	H	H	H	O	488	369	47	904	14.6	0.201	2.796	127.21	2.675	5.817	1.468
3	H	H	H	O	O	450	350	59	859	16.9	0.233	2.250	120.63	2.396	3.234	1.267
4	H	H	O	O	O	323	226	34	583	13.3	0.264	1.708	90.33	1.793	3.272	1.145
5	H	O	O	O	O	236	222	29	477	13.0	0.264	1.708	73.88	1.582	3.156	0.794
6	H	O	H	H	H	381	402	38	821	14.7	0.219	3.248	114.57	2.296	4.661	1.432
7	H	O	O	H	H	306	255	41	602	15.0	0.282	2.371	93.26	1.784	3.056	1.424
8	H	O	O	O	H	290	271	53	614	16.3	0.220	3.184	91.11	1.795	2.796	1.195
9	H	H	O	O	H	419	154	37	710	14.0	0.240	2.820	90.75	1.814	2.714	1.295
10	H	H	O	H	H	476	471	69	1016	14.2	0.260	3.158	135.54	2.776	4.723	1.285

Consequently, potassium is required by the sugar-beet both during leaf-formation and at the period of root-development. This is borne out by the figures quoted for the experimental run No. 5 in which the sugar-beet, withheld from its potassium supply 55 days after planting and kept so during 15 days, gave a yield inferior to that obtained in any other run. A particularly poor development was observed in such plants as were deprived of potassium at the time of the appearance of the 4th pair of leaves.

A subsequent transfer of the plants to a full nutrient solution could not compensate for potassium starvation up to the end of the vegetating period, and the plants remained behind the controls in their development. This went once more to confirm that sugar-beet requires an uninterrupted supply of nutrient substances.

If the period during which the plant is withheld from potassium is protracted to 30 days, the yield is still more badly affected by it; and it becomes still more evident that sugar-beet is absorbing potassium most intensely at its earliest age. Potassium starvation has a slighter effect when applied to the plant during the subsequent development stages, than if applied at the period of appearance of 4—7 pairs of leaves. In the potassium nutrition of the sugar-beet this time-interval appears to be a critical one, both total yield and sugar content being strongly affected by potassium deficiency.

When the plant is subjected to long potassium starvation, as well as if this element is excluded from its diet during the critical period, its leaves begin to die away, and the accumulation of sugars in the root suffers inhibition. In the case of potassium deficiency the sugar-beet shows disturbances in the normal translocation of assimilates (sugars) to the root. From a comparison of yield, records for runs 1, 2, 3, 4 and 5, it may be seen that sugar-beet thrives better on potassium throughout the vegetation period. On the other hand, the analysis of the plants for their potassium content showed them to absorb the largest amount of potassium at the early period of vegetation, which coincides with the time of an intense increase in the leaf-area.

By comparing the experimental runs 1, 6, 7, 8 and 5 no direct relation could be observed between the decrease in the yield and the period during which the plant was kept at a full nutrient solution. If sugar-beet is kept on potassium diet during a short time, after having absorbed a small quantity of this element the plant is found to turn it to high profit. Both absolute and relative sugar-content are found to decrease according as the period of potassium starvation is protracted.

The content of colloids and «harmful» nitrogen in the root of sugar-beet is also affected by the period during which the plants are kept on potassium diet.

By excluding potassium from the nutrient solution the uptake of nitrogen and phosphorus by the plant is also affected: along with an increase in the period of potassium starvation, the plant is found to consume less phosphorus, which is borne out by the respective figures from runs 1—5.

If sugar-beet is withheld from potassium diet at different development stages, the following picture can be observed: the plant has absorbed a considerable amount of potassium during the period between planting and formation of the 4th pair of leaves (runs 1—5). When withheld from potassium for varying time-intervals at this period, the plant suffers no sharp decrease in its potassium content.

If potassium is excluded from the plant's diet for varying time-intervals during the medium stages of development, the plant is found to have the lower potassium content, the longer it was withheld from potassium.

If after the appearance of 7 pairs of leaves sugar-beet is transferred to nutrient solutions from which potassium was excluded, the accumulation of this element by the plant is dependent upon the time during which it had been kept on potassium diet.

The absorption of phosphoric acid has been found to be entirely dependent upon potassium uptake.

On the basis of yield records and analytical data the following conclusions are to be drawn:

1. If potassium is excluded from the diet of sugar-beet during early periods of the plant's development (4—7 pairs of leaves), the result is a reduced yield and an impaired quality of the crop.

2. As a critical period in the plant's potassium uptake should be considered the period of intense sugar accumulation (appearance of the 7—10 pairs of leaves).

3. Potassium deficiency in the nutrient medium has a negative effect upon the size of the leaf-area.

4. The curtailing of the period during which sugar-beet is kept upon potassium diet results in a reduced sugar output.

5. If potassium is excluded from the plant's diet during its growing period, the amount of colloids in the cell-sap of sugar-beet is increased.

6. Potassium deficiency during the second half of the vegetation period results in an increase in the percentage of soluble nitrogen.

7. The maximum uptake of nutrient substances by the sugar-beet does not coincide with the critical periods of potassium nutrition.

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PLANT PHYSIOLOGY

A CONTRIBUTION TO THE KNOWLEDGE OF THE BREAKING OF WINTER DORMANCY IN BUDS OF WOODY PLANTS

By A. A. RICHTER, Member of the Academy, and T. A. KRASNOSSELSKAYA

We have shown elsewhere (1) that it is possible to break profound winter dormancy in the buds of woody plants by stimuli applied not directly to the resting organs, not locally, but at some distance from them. In other words, the impulse necessary for awakening resting buds from their profound winter dormancy can be transmitted along the shoot both upwards and downwards. In a number of our experiments as a source of such impulses served buds grafted on to dormant shoots.

These facts enabled us to arrive at a conclusion as to «the existence of certain products of vital activity of the type of hormones, inducing profound alterations in the physical-chemical state of the resting cells» (1). This conclusion was, of course, a hypothesis standing in need for experimental verification. Further lines of an experimental approach to the verification of this hypothesis are suggested in the present note.

Lime-tree and ash were selected for the experiment as two species showing under the conditions prevailing during winter in Moscow the most stable indices of profound winter rest. When kept for a long time in a warm room (at 15—17°C) the shoots of the two species remained in full rest, their buds showing not the slightest indices of development until the end of February and the beginning of March.

By applying warm baths after Molisch, the buds were made to unfold as soon as within a few days, which was followed by the formation of green leaves.

Within the first days after they had left the state of dormancy, these «awakened» buds were used as a source of the alleged factor of breaking dormancy. In order to avoid all conditions connected with vital processes and constructions, we had resort to the simplest method for destroying the living tissues of the awakening buds: two buds were ground in a mortar with 2 cm^3 of water until all the tissues were destroyed and converted into structureless pulp. This material was employed for experiments which were arranged as follows. A cut 3 cm long was made through the bark along the dormant branch; two perpendicular cuts, each of them half way around the branch, were made at the ends of the first cut. The bark was then cautiously lifted, and 2—3 drops of the pulp introduced under it.

After this the bark was brought into contact with the wound surface moistened with the pulp, and the operated place fixed with insulating ribbon. Side by side with the branches operated in the above manner two kinds of control branches were placed in a container with water: 1) non-operated ones and 2) branches operated in the same way as the experimental ones, save for their wounds moistened with pure water instead of the pulp.

The results of the treatment of branches with pulp made of the awakened buds became manifest after 5—7 days. While the branches which had undergone no treatment or had been treated with water only remained inert and failed to unfold their buds, those which had received the pulp from unfolding buds displayed first a clearly expressed swelling of buds throughout the shoot, followed by the unfolding of leaves. The shoot awakened from its profound dormancy and started growing.

These most simple experiments which demand no complicated apparatus and can easily be reproduced seem to show with full certainty that the main part in the process of the breaking of dormancy of the woody plants is played by stimulating agents of the type of hormones.

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PLANT PHYSIOLOGY

PHOTOPERIODISM OF INDIVIDUAL PARTS OF THE LEAF, ITS HALVES

By M. ČH. CAJLACHJAN

(Communicated by A. N. Bach, Member of the Academy, 31. XII. 1944)

The phenomenon of photoperiodism which had been discovered by Garner and Allard([10]) became a subject of the studies of many workers because of immense importance for the plant's growth and development. One of the most interesting facts discovered in the course of these studies is that the receptor of the photoperiodic effect is the leaf ([1], [4], [5]). A deeper insight into the mechanism of the photoperiodic reaction has led to experiments in which individual leaves, instead of the whole foliage of the plant, were subjected to a special light regime. In this way new regularities underlying the relations between leaf and shoot could be born out ([2], [6–8], [11]).

In the clearest form these relations were brought out by our experiments ([9]) which showed that a shoot either quickly passed into flowering and fructification, or remained in the stage of vegetative growth, according to whether it was exposed to short or long photoperiods.

In order to compare the relative susceptibility to the photoperiodic action of individual parts of the leaf, it was necessary to provide for a differential light regime within a single leaf blade.

With this aim in view we carried out special experiments with *Perilla nankinensis* (a short-day plant) in 1942, at the greenhouse of the Department of Plant Physiology and Anatomy of the Erevan State University. Additional experiments were conducted in 1944 with the same species and with spinach, variety «Vitofik» (a long-day plant) in the greenhouse of the Institute of Plant Physiology of the Academy of Sciences of the USSR.

In the course of experiments of 1942 it has been found that by keeping the longitudinal half of the leaf alone at a short (9-hour) day regime flowering of the axillar shoot in perilla is affected about in the same manner as when the whole leaf is subjected to the treatment. When cross-halves were shaded, a sharp arrest in the flowering of the shoots ensued in those cases where the apex of the leaf was kept at a short and the base, at a long day; the inhibiting effect is considerably less with a reversed arrangement of the light regime.

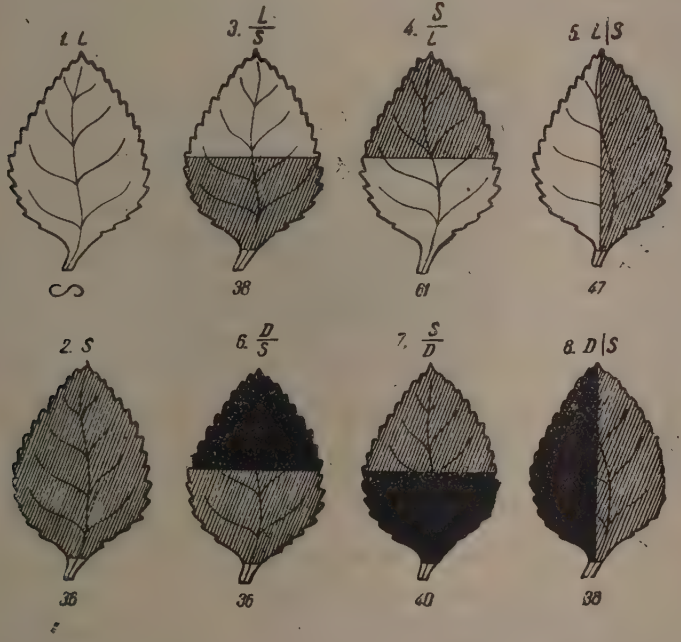
In the 1944 experiments seeds of perilla were planted in boxes on the 5th of May, at the Institute of Plant Physiology; on the 5th of June the young plants were transferred to 18 cm pots and kept until the beginning of the experiments at natural long day. Spinach was sown on the 6th of June in 18 cm pots and kept at short 10-hour day. At the middle of July (17.VII) the plants were trimmed in the following manner: all the leaves and shoots, except for the apical bud, and a single large fully developed leaf at the fifth node (from the base) were removed from each plant of perilla; from each rosette of spinach all leaves except

for one biggest leaf of the growing cone in the centre of the rosette, were also removed.

The experiments with perilla were arranged on the following scheme.

A. Basic Variants: 1) entire leaf kept at long natural day (L); 2) entire leaf at short day (S); 3) base of leaf received short, apex—long day $\left(\frac{L}{S}\right)$; 4) base received long, apex—short day $\left(\frac{S}{L}\right)$; 5) left longitudinal half of the leaf received long day, the right one—short day (L|S); 6) the base received short day, the apex remained in continuous darkness $\left(\frac{D}{S}\right)$; 7) the base was kept in darkness, the apex received short day $\left(\frac{S}{D}\right)$; 8) the left longitudinal half of the leaf was kept in darkness, the right one received short day (D|S).

All these variants are given on the adjoining scheme.



B. Additional «Sliding» Variants: 9) first 6 days $\frac{L}{S}$, the next 6 days $\frac{S}{L}$, *etc.* $\left(\frac{L}{S}+\frac{S}{L}\right)$; 10) L|S+S|L; 11) $\frac{D}{S}+\frac{S}{D}$; 12) D|S+S|D. The experiments with spinach were conducted according to the scheme of the first five basic variants for perilla.

Each variant was conducted in double replication. Shading of the leaves was achieved by means of cases made with heavy dark-blue paper. For creating short day the cases were put on at 5 o'clock p. m. and taken off at 9 a. m. next day, so that the light period of the day was equal to 8 hours; in the case of continuous darkness the cases were kept on throughout the whole period of experiments. The experiments with perilla were started on the 21th of July and finished on the 20th of September; the experiments with spinach were started on the 20 th of July and finished on the 20th of August. In the «sliding» variants the reversal of the light regime was done six times, *viz.*, on the 27th of July, 2nd, 9 th, 15th, 21th and 28th of August and on the 9th of September. Every second or third

Development of Shoots as Affected by Differential Light Regime (Different Length of Illumination of Halves of Leaf)

Number of exper.	Variant	Budding		Flowering	
		date	number of days since beginning of exper.	date	number of days since beginning of exper.
	I. Perilla				
	A. Basic variants				
1	L	none	—	none	—
2	S	26. VIII	36	2.IX	43
3	$\frac{L}{S}$	28. VIII	38	11.IX	52
4	$\frac{S}{L}$	20. IX	61	none	—
5	L|S	6. IX	47	»	—
6	$\frac{D}{S}$	26. VIII	36	4.IX	45
7	$\frac{S}{D}$	30. VIII	40	10.IX	51
8	D|S	28. VIII	38	9.IX	50
	B. «Sliding» variants				
9	$\frac{L}{S}+\frac{S}{L}$	11. IX	52	none	—
10	L|S+S|L	15. IX	56	»	—
11	$\frac{D}{S}+\frac{S}{D}$	28. VIII	38	8.IX	49
12	D|S+S|D	26. VIII	36	8.IX	49
	II. Spinach				
	Shooting				
1	L	1. VIII	12	—	—
2	S	none	—	—	—
3	$\frac{S}{L}$	1. VIII	12	—	—
4	$\frac{L}{S}$	12. VIII	23	—	—
5	L|S	1. VIII	12	—	—

day all the newly appearing shoots and leaves were removed in order to keep the scheme in its original pure form till the experiments were brought to the end.

Data on the development of shoots in perilla and spinach in all the experiments are summarized in the table.

The data of the table show that, like in the previous experiments in perilla, there is a sharp difference between the variants $\frac{L}{S}$ and $\frac{S}{L}$; when the base of the leaf is exposed to short day $\left(\frac{L}{S}\right)$, the transition of the respective shoot to repro-

ductive development is but slightly behind the budding of the shoot whose leaf remains entirely under short day conditions (S); when the apex of the leaf is exposed to short day $\left(\frac{S}{L}\right)$, the lagging is a very sharp one. The zone of the long day (base of the leaf) which separates the shoot from the short-day zone (apex) appears to be a sort of barrier which strongly prevents the transmission of the photoperiodic influence from apex to shoot. When the long-day and the short-day zones are situated longitudinally (L|S), the arrest in development of the shoot is much less pronounced, although sufficiently expressed all the same, in conformity with analogous observations made earlier by Moshkov (3).

A similar barrier is created also in the leaf of spinach when its base is kept at short day and the apex at a long day $\left(\frac{L}{S}\right)$, for it is just in this case that the formation of flowering shoots is sharply laggiyng in contrast to the other variants: S, $\frac{S}{L}$ and S|L.

In perilla this barrier is removed when the base of the leaf is kept, instead fo long day, in full darkness. Indeed, the budding of the shoots in the variant $\frac{S}{D}$ differs but slightly as regards its date from that of the shoots whose leaves receive shading of the apical half $\left(\frac{D}{S}\right)$ or of the longitudinal half (D|S). Unlike the long day, darkness does not hinder at all the transmission of the photoperiodic effect.

Besides, the behaviour of the shoots in the variants with continuous darkness $\left(\frac{D}{S}, \frac{S}{D} \text{ and } D|S\right)$ give a full answer to the question as to the causes responsible for the inequality of the apical and basal halves of the leaf; this question arose as a result of the first experiments. It is not the difference in the susceptibility to the photoperiodic effect which determines the different response of the basal or apical halves of the leaf to short or long day, but rather the difference in the spatial conditions of the transmission of this effect.

In the «sliding» experiments involving alternating exposures of either half of the leaf $\left(\frac{L}{S}+\frac{S}{L}, L|S+S|L\right)$, as it should be anticipated, the development of the shoots is found to be behind that in $\frac{L}{S}$ and L|S. However, in the variants $\frac{D}{S}+\frac{S}{D}$ and D|S+S|D there is almost no lagging in the development of the shoots as compared with $\frac{D}{S}$ and D|S, while in the latter instances the short day acts all the time upon the cells of one and the same area, in the former instances the cells of a given area receiv› short day and darkness alternately. This points to the fact that when leaves of a short-day plant are kept in darkness, not only such conditions are created as permit of a labile transport of the photoperiodic effect, but those processes are also in progress which had been induced by short day.

The processes induced by short day in the short-day plants (perilla) and by long day in the long-day plants (spinach) are those of the formation of the hormones of flowering (4). The hormones of flowering which arise in the apical half of the leaf encounter in their migration the hindering effect of the long-day zone (the base of the leaf in the case of perilla) and of the short-day zone (in the case of spinach).

This arrest can be explained neither by dispersion of the flowering hormones in the cells of the base of the leaf nor by the extension of the distance they have to pass, for darkness (in the case of perilla) does entirely remove this obstacle.

The solution of this problem should be sought for in the study of the physiological nature of this peculiar barrier, in the relations between the processes arising under long- or short-day conditions. The method of differential light regime

imposed to the individual parts of the leaf, as described above, may prove of great help here, since it permits to bring as close as possible the short-day zone and the long-day zone of the leaf.

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ENDOCRINOLOGY

EFFECT OF INCREASED FECUNDITY IN SWINE INDUCED WITH THE AID OF THE SPM GONAD STIMULATING HORMONE

By B. M. ZAVADOVSKY

(Communicated by I. I. Schmalhausen, Member of the Academy, 27. VII. 1944)

In our preceding notes ([1]) we reported on the positive effect of artificially induced heat, successful coverings and on the insuing farrow in swine injected with SPM (serum of pregnant mares) of the I and II group. This effect could not, however, be duly evaluated either economically, or from the point of view of the applicability of the method suggested in zootechny until adequate data were available as to the number of young in the litters of the experimental animals and to the condition of the litter, the status of the young and their weight.

Corresponding records were made by us for the experimental sets of animals of the Lusin hog-breeding Soviet farm belonging to the Omsk Trust for Hog Industry. In order to make comparable the conditions of keeping, feeding and tending, such young of both experimental and control sows were inspected as were tended by the same swine-herds. The results of the records are summed up in the table.

	Number of dams	Average number of young per litter	Average weight of young pigs by 1st month, in kg	Weight of young pigs by 2nd month
Experimental Set* Lot 1				
Experimental dams	5	9.6	6.4	—
Controls	9	8.5	5.8	—
Lot 2				
Experimental dams	9	10 (11)	5.0	—
Controls	4	9	5.0	—
Experimental Set II**				
Experimental dams	16	9.5	5.0	13.2
Controls	13	8.2	5.5	13.2

* Covering on November-December 1942; farrows on March, 1943.
** Covering on March, 1943; farrows on June-July, 1943.

In several cases the experimental pigs were weighed directly upon farrow; the figures obtained have been found to average 1.0 kg, both in experiment and control.

Allowance being made for a considerable underfeeding accounted for by war conditions, these figures should be considered as being quite satisfactory. It should be kept in mind that under normal feeding conditions the weight of newborn pigs of a white English sow fluctuates, as a rule, between 1.0 and 1.2 kg.

The young pigs produced by the experimental dams differed from those of the controls neither in their weight nor in the general status and health, save for some negligible fluctuations to either side. At the same time a well-pronounced effect of increased fecundity was observable in all experimental sets as compared to the controls: the litters produced by dams treated with SPM invariably exceeded those produced by the control animals by one live, viable pig. This advantage gains in importance still if the figures for stillborns are taken into account. Thus, within lot 2 cases of striking individual fecundity could be observed. Dam No. 3070 which on the 25th day of suckling on the 20th of November was treated with SPM serum of the I group, had never produced more than 6 young in the case of natural covering. On the 20th of March this sow produced a litter comprising alongside of 12 live pigs 4 stillborn. Dam No. 6298 received on June 22th, 5.0 cm³ of SPM «Borovoje» (group I); on September 8th this sow produced a litter of 5 young. After having been treated with SPM this dam produced a litter consisting of 10 young alive and 2 stillborn. The experimental litter of dam No. 6046 consisted of 12 viable pigs; the one of dam No. 1230, of 13 pigs alive and viable; dam No. 5963 gave birth to a total of 14 young, of which 11 were viable and 3 stillborn.

If all the stillborns be taken into account, within lot 2 we obtain an average of 11 young per experimental dam, which makes an excess of 2 young as compared with the litters of the controls. A stable effect of steadily increasing fecundity is thus traceable throughout the experiments. It should be pointed out that under the conditions of food deficiency stillborn are of no infrequent occurrence in the practice of hog-breeding and should therefore by no means be attributed to the effect of the SPM.

It should be pointed out likewise that in none of our earlier experiments involving the application of prolane from the urine of pregnant women did we obtain a similar effect of increased fecundity.

A close analysis of the conditions which contributed to the effect of increased fecundity obtained in our experiments permits of the following conclusions:

1. Most of the swine-dams remarkable for their high productivity were subjected to treatment with the SPM either after 60 days after the farrow, or, as in the case of dam No. 3070, by the 25th day of suckling, which periods are closely coinciding with those of setting in of natural heat and ripening of follicles. It is obvious that the effect of increased fecundity can regularly be induced in the case where the supply of gonadostimulators from without is overlapping the natural processes of the ovary's passing to the active stage, calling forth the ripening of a certain number of additional follicles and the ripening of adequate egg-cells.

Let us remind in this place that a similar concomitant effect of increase in fecundity was observed for the first time in 1935—36 while applying the SPM with the purpose artificially to induce heat in sheep. Corresponding experiments were carried out under my personal guidance in Ascania Nova, and the effect obtained is at the present time being turned to advantage by A. I. Lopirin and M. M. Zavadovsky in their method for inducing experimental superovulation in sheep. The regularity with which the effect is observed to reoccur gives every ground to expect that it can be stabilized in hog as well.

2. All dams which had proved to be high producers (litters containing above 11 pigs) were treated with the SPM of the I group. The serum of the II group (from the mare «Katerinushka») which gave the best effect in point of induced heat and successful fertilization, was found to produce no changes in the number of offspring as compared to the controls, the average number being 9 per litter in either case.

We are thus brought to believe that the SPM of the II group compares favourably with that of group II as to the effect of increased fecundity.

The effect of the SPM of the I group here described, *i. e.* a well-pronounced prevalence of the follicle-stimulating factor «A», is in good agreement with the theory which has been developed by the present author during a number of years.

According to this theory, it is just the follicle-stimulating factor «A» that plays a decisive part in the process of ripening and growth of the follicles and of the eggs they contain; whereas the effect of factor «B» is to stimulate ovulation in these follicles when mature, and the formation of *corpora lutea*.

The brilliant studies by the American endocrinologists Van Dyke and Evans ([5],[6]) went fully to substantiate those theoretical concepts of ours. Until recently both authors were the leaders of that influential group of endocrinologists who were advocating the theory making a single agent responsible for the effect of the stimulation of sex-glands. These writers were inclined to explain any incongruence in the work with corresponding preparations by quantitative differences in the doses applied only.

In their latest work, however, both Van Dyke and Evans have independently of each other changed their former attitude, being now thoroughly proficient in the concept of two gonad stimulating agents. Van Dyke has succeeded, moreover, with the aid of his collaborators to isolate in their pure state protein preparations from the pituitary bodies of hog and sheep. These preparations are identified by those writers with the luteinizing hormone, termed factor «B», according to our terminology.

Another hormone of proteinic nature has laso been isolated in nearly a pure state by the same two experimenters working independently of each other, *viz.* factor «A»,stimulating the formation of follicles.

The above-named writers have succeeded in isolating the active principles of either of these hormones which made it possible to shed more light upon their respective rôle and their interaction in propagation. Thus, according to their data, factor «A» is a hormone whose work it is directly to determine and control the processes of ripening of sex-cells in the two sexes, *i. e.* the ovogenesis in the ovary and the spermogenesis in the testicles. In its pure form, though stimulating the growth of the follicles and promoting the ripening of the egg-cells, factor «A» is incapable, however, to stimulate the production of folliculine and the setting in of heat.

In order adequately to stimulate not only the generative functions, but the endocrine functions as well, *i. e.* in order that the follicles be able to produce the oestrogenic hormones, there is necessary a certain amount of the second component, *i. e.* of the gonad stimulator which is responsible for the production of the hormone contained in the *corpora lutea*.

Thus, the part played by factor «B» appears to be much more important and the sphere of action much wider: it seems to control not only the ultimate stage of the ripening of follicles, ovulation and production of the hormone of *c. luteum*, but also to regulate, jointly with factor «A», all the preceding stages of the production of the oestrogenic sex-hormones in earlier stages of the ripening of follicles.

The name coined for this gonad stimulator by Van Dyke and Evans is ICSH (Interstitial Cell Stimulating Hormone), which means a hormone whose effect it is to stimulate the interstitial or endocrine tissue of both ovary and testicles.

All that has been said above, as well as the experimental data here reported, are found to be in fair agreement with our principal theoretical conclusions and with the results of our experiments. The latter have shown that in order to obtain the highest activation effect of both generative and endocrine functions of the follicular apparatus of the ovary, it is necessary that the two gonad stimulants be present in the SPM in a definite ratio. The optimal ratio for the purposes of inducting artificially heat is offered by SPM preparations of the II group,

while for the purpose of stimulating fecundity in hogs we also find the optimal combination, and perhaps largely, in the SPM of group I, which is the highets in the hormone whose effect it is to stimulate the generative functions of the ovary (2).

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MICROBIOLOGY

ON THE TAXONOMY OF NITRIFYING BACTERIA

By A. IMŠENECKI

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In studying cellulose decomposing bacteria isolated from some soils of the Transvolga region, in the year 1933, I found among them typical myxobacteria which intensely decomposed cellulose. This circumstance directed my attention towards the *Myxobacteriales*. My subsequent researches on the biology of these microorganisms have shown that species greatly differing in their physiology were to be found among myxobacteria. It seemed possible to assume that forms partaking in the various transformations of carbon and nitrogen had arisen in the order of *Myxobacte-riales* as a result of divergence.

The morphology and taxonomy of myxobacteria are little known to the wider circle of microbiologists, therefore I had recourse to the descriptions of microbes available already, striving to find among them species belonging to the myxobacteria and erroneously classed with other systematic groups. Particular interest was offered in this respect by bacteria which had been previously described as new agents of the process of nitrification.

It is known that the problem of the biological oxidation of ammonia has been solved in a brilliant way by our countryman Winogradsky (1). He has isolated in pure culture a bacterium which oxidated ammonia into nitrous acid, and gave it the name of *Nitrosomonas*.

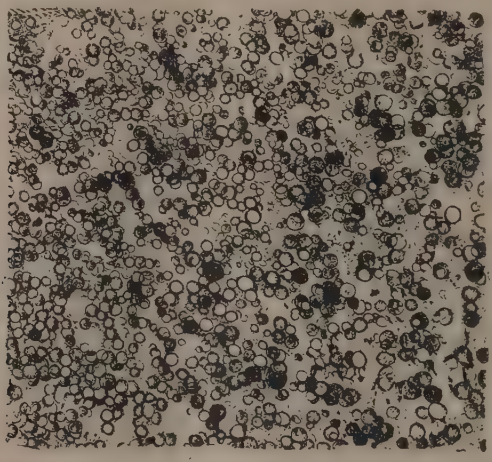
For forty years *Nitrosomonas* has been the only chemotrophic organism, the nitrifying ability of which was universally acknowledged. The attempts to show that there exist besides it heterotrophic or myxotrophic nitrifying bacteria could not withstand experimental testing and are now but of purely historical interest.

Such was the situation until 1928, when the Swedish microbiologist Romell (2) working in the laboratory of Winogradsky discovered in forest soils peculiar nitrifying bacteria which produced a zooglea growth and large spheroid forms (cysts). Later similar organisms were isolated from soils by Winogradsky himself, who, desiring to emphasize the ability of these bacteria to form cysts, gave them the name of *Nitrosocystis*. In studying a soil specimen brought by Prof. B. L. Issachenko from Nova Zembla, Winogradsky found in it also nitrosobacteria of a new kind, morphologically resembling spirochaetes. He classified them as a new genus: the *Nitrosospira*. Still later Helen Winogradsky, studying the nitrifying organisms of slimes, found a number of new nitrificators. Some of these she classified with the *Nitrosocystis*, while several other species which grew as mucous zoogleas she united into a new genus, the *Nitrosogloea*.

These researches, consequently, record the existence of a considerable number of new nitrifying microbes, differing greatly in their biology from the *Nitrosomonas* which had been known for so long.

The descriptions of the morphology of the new nitrifying bacteria, contained in these studies, as well as the photographs supplementing them convinced me that the authors had dealt with cultures of myxobacteria. This induced me to isolate similar organisms and to establish definitely their systematic position.

In these investigations use was made of the technique earlier applied by Winogradsky. 2 cm^3 of a medium of the following composition: K_2HPO_4, 0.5 g; $MgSO_4$, 0.3 g; NaCl, 0.3 g; $FeSO_4$, 0. 02 g; $MnSO_4$, 0.02 g ; tap water, 200.0 g, were poured on the surface of silica gel which had solidified in Petri dishes. The medium was dried a little, after which 1 cm^3 of a 5 per cent solution of $(NH_4)_2SO_4$ was poured on it, and it was again dried. After this the surface of the medium was covered with a chalk suspension. The dishes were again subjected to drying,



Fruiting bodies of myxobacterium *Sorangium symbioticum* in zone of nitrification. Magn. 95×

small clots of soil were laid out on the surface of the gel and the dishes were incubated at 25°C.

In one of these experiments clear zones showing the dissolution of chalk appeared around the soil clots after three weeks' incubation. The soil specimens in this case originated from a karagach grove in Kirghizia (in the vicinity of Frunze.) The zones were 0.5—1 cm wide and were sharply outlined. The surface of the denudated gel was covered with slightly mucous sparse film of a yellowish-brown colour. At a small magnification one could easily discern within the zones of chalk dissolution pale-yellow fruiting bodies of a rounded shape (cf. figure).

Usually these spheroid forms are larger in size near the soil clots, while nearer to the margin of the zone they are smaller. The size of the fruiting bodies varies between 18 and 40 μ. The bodies are composed of rounded or polygonal cysts lying closely together, and consequently the body itself has the shape of a mulberry. The cysts in their turn consist of smaller cysts (3—4 μ in diameter) of polygonal, oval or spheroid shape. The fruiting bodies have no solid envelope and can be easily crushed with the help of a cover-slip, after which it becomes evident that the small cysts consist of accumulations of short rod-shaped bacterial cells.

Bits of gel taken from the zones of chalk dissolution gave a sharply positive nitrite reaction (with the Tromsdorff reagent). The formation of nitrites and the disappearing of ammonia in the course of prolonged cultivation of the microorganism described—both suggested that nitrification was going on in reality.

The morphology of the fruiting bodies and the structure of the cysts themselves left no doubts that the organisms developing on the gel should be classed with the myxobacteria and belonged to the genus *Sorangium*. In its outward

appearance this species approaches the nitrifying bacteria described by Romell and Winogradsky.

The dissolution of chalk on gel plates impregnated with salts, *i. e.* in a medium lacking every organic ingredient, formed a weighty argument for considering this myxobacterium a nitrificator. But this fact taken apart could not solve the question. Accordingly, I undertook transfers which should either lead to the isolation of *Sorangium* in pure culture, or prove that this species is chemotrophic and unable to grow in common media. With this aim in view, the film from the zones of chalk dissolution was transferred on a series of Petri dishes with the medium commonly used for the cultivation of myxobacteria. Its composition was as follows: 20 per cent potato infusion, 100 cm^3; peptone, 0.5 g; agar-agar, 2 g.

Transfers grown in this medium showed that *Sorangium* forms on it shiny round yellow colonies. They are slightly mucous, convex, smooth and possess an even margin. After 5—6 days of growth grouping of cells can already be observed in the colonies, and the formation of cysts begins; later on these cysts agglomerate giving rise to spheroid fruiting bodies. Fruiting bodies and cysts are particularly numerous in old giant colonies of *Sorangium*, the age of which reaches 45—50 days.

The colour, size and structure of the fruiting bodies, as well as the abundant growth of this species observed in transfers from the zones of nitrification, does not leave any doubt that the *Sorangium* developing on the gel and the strain growing on potato-agar are one and the same species.

Thus, the possibility of the existence of a chemotrophic myxobacterium was eliminated. Only the alternative assumption remained, *viz.* that the *Sorangium* developing on the gel is a myxotrophic organism.

A pure culture of *Sorangium* was grown proceeding from a single cell and studied. It was found that in 24—48-hour cultures in solid media the vegetative cells of *Sorangium* have the shape of rods 0.5×2.5 μ in size, sometimes slightly arched. The cells show a very active motion, but flagella could be discovered neither with the help of an electrone microscope nor by means of staining according to Löffler. The ability of motion in the absence of flagella is a specific characteristic of myxobacteria, and it is probably of reactive nature. Vegetative cells of *Sorangium* gradually contract with the ageing of the culture, they become shorter and somewhat thicker at the same time. In 5—6-day cultures their measurements are 1.0×1.2 μ. The shortened rods group and form cysts which in their turn agglomerate into fruiting bodies described above.

The isolated *Sorangium* grows readily in usual broth, ordinary media: meat-peptone agar, wash-agar, wash and potatoes. Growing in meat-peptone or potato gelatine it does not liquefy it. Transferred to a mineral medium with glucose and varying sources of nitrogen, it grew only in the medium containing peptone. In media containing albumen, $(NH_4)_2SO_4$, KNO_3 or $NaNO_2$ it did not grow. Thus, in its physiological characteristics the *Sorangium* isolated by us differs but little from a number of other myxobacteria.

The elucidation of the ability of *Sorangium* to nitrify was of principal significance. Therefore it was tested by means of various methods. In one series of experiments the culture of *Sorangium* was transferred from potato-agar to the surface of a gel plate impregnated with a special medium for nitrificators, and the cultures were incubated at 25° C for two months. In another series myxobacteria were seeded into a series of flasks with a liquid medium for nitrificators. Finally, in a third series «acute» experiments in nitrification were arranged. For this purpose a dense suspension of *Sorangium* cells, washed off from the surface of a solid medium, was introduced into a liquid medium for nitrifying bacteria.

The result of every test was negative. The cells would not multiply, there appeared no zones of dissolution of the chalk on gel plates, ammonia was not oxidated and no nitrites could be detected in any of them. Consequently, the isolated myxobacterium possesses no nitrifying ability and is a typical heterotroph.

The question arose naturally. How to explain that *Sorangium* develops on

silica gel and is found in the zones of chalk dissolution? The answer to this question is obtained by observing under the microscope the chalk-covered area around the zones of nitrification. Cells of the classic nitrifying agent *Nitrosomonas* can be regularly discovered in preparations from these areas of the gel plates. These are round, light refracting cocci lying perfectly separated and intensely stained by basic stains. At the very margin of the zone of nitrification one find palely stained agglomerations of myxobacterial cells, among which single cells of the nitrificator may be seen. Thus, *Nitrosomonas*, being a chemotroph, begins, first, its development on the gel plate and progresses gradually forwards leaving behind a bacterial mass, *i. e.* organic substance, ready for use. After the chemotroph begins to develop the myxobacterium *Sorangium* which utilizes the cells of *Nitrosomonas* as a source of nutrition. It is possible that cell autolysis of the nitrificator aids the development of myxobacteria. The spatial relations of the two species of microbes are such that *Nitrosomonas* may be termed «avanguard» organism for the *Sorangium* which follows it closely.

Foreign species are always present in enriched cultures of nitrifying bacteria, but these are usually vulgar accompanying bacteria. Contrariwise, stable symbiotic relations have arisen between the chemotrophic *Nitrosomonas* and the heterotrophic *Sorangium*, relations which remain unchanged in the course of numerous transfers. The nitrificators provide the organic substance which is necessary for the growth of the myxobacteria, and it seems quite probable that in their turn the myxobacteria by their mucous agglomerations preserve the cells of *Nitrosomonas* from desiccation and from the action of extraneous factors. Accordingly, I have given to the species of myxobacterium isolated by me the name of *Sorangium symbioticum*.

The results of my researches may be summed up as follows.

Zones of chalk dissolution were obtained and accumulation of nitrites was established in silica gel plates, impregnated with a special medium for nitrifying bacteria, on which small clots of a soil had been seeded. In these zones were discovered spheroid bodies (cysts) characteristic of the new species of nitrifying bacteria, described by Rowell, Winogradsky and H. Winogradsky by the name of *Nitrosocystis*, but not isolated by them in pure culture. It was established by means of morphologic analysis that these spheroid bodies are the fruiting bodies of a myxobacterium belonging to the genus *Sorangium*. The pure culture of *Sorangium* proved to be heterotrophic and unable of nitrification. The development of myxobacteria in media used for nitrificators is explained by their symbiosis with the genuine nitrificator—*Nitrosomonas*—which can be regularly found in such cultures. The nitrifying bacteria synthetize organic substance, the myxobacteria utilize it.

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